Oikos OIK-07188

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Supplementary results and analysis for ”Infection state can affect host migratory decisions”

A Complete analytical expression of ESS

As stated in the main text, the matrix χ relating the number of susceptible and infected individuals from one generation to the next is defined as

$$\begin{bmatrix} S \\ \\ \\ I \end{bmatrix}_{\tau+1} = \begin{bmatrix} (1+\Delta\phi_S)[(A(1-\theta_S)+B\theta_S+C\theta_I)] & (1+\Delta\phi_S)D\theta_I \\ +\Delta\phi_I(G\theta_I+E(1-\theta_I)+F(1-\theta_S)) & +\Delta\phi_I(J\theta_I+H(1-\theta_I)) \\ \\ G\theta_I+E(1-\theta_I)+F(1-\theta_S) & J\theta_I+H(1-\theta_I) \end{bmatrix} \begin{bmatrix} S \\ \\ \\ I \end{bmatrix}_{\tau}$$

where

$$
\begin{aligned}
A &= \sigma(1-c_R)e^{-\beta(T_1+T_2)} \\
B &= \sigma(1-c_M)e^{-\beta T_1} \\
C &= \sigma(1-c_M)(1-e^{-\beta T_1})(1-e^{-\gamma T_2}) \\
D &= \sigma(1-c_M)(1-e^{-\gamma T_2}) \\
E &= \sigma(1-c_I)(1-e^{-\beta T_1}) \\
F &= \sigma(1-c_I)(1-e^{-\beta T_2})e^{-\beta T_1} \\
G &= \sigma(1-c_M)(1-c_I)(1-e^{-\beta T_1})e^{-\gamma T_2} \\
H &= \sigma(1-c_I) \\
J &= \sigma(1-c_M)(1-c_I)e^{-\gamma T_2}.
\end{aligned}
$$

We then use the Jury criteria for stability to determine the the value of (θ_S , θ_I) at equilibrium. We find that the Jury criterion Tr[$\chi(\theta_S{}^{'}, \theta_I{}^{'})$] - Det[$\chi(\theta_S{}^{'}, \theta_I{}^{'})$] < 1 is the most sensitive indicator of the system's stability and if it is not violated, then none of the other criteria are violated as well. Additionally for a stable system, the dominant eigenvalue of the system (λ_1 henceforth) is equal to 1. For such a system, we can show that Tr[$\chi((\overline{\theta_S}, \overline{\theta_I})$]-Det[$\chi((\overline{\theta_S}, \overline{\theta_I})$] = 1. Therefore, we can say that a migration strategy ($\overline{\theta_S}, \overline{\theta_I}$) is evolutionarily stable to invasions by a mutant with any other strategy ($\theta_S{}^{'}, \theta_I{}^{'}$) if

$$
Tr[\chi(\theta_S{}^{'}, \theta_I{}^{'})] - Det[\chi(\theta_S{}^{'}, \theta_I{}^{'})] < Tr[\chi(\overline{\theta_S}, \overline{\theta_I})] - Det[\chi(\overline{\theta_S}, \overline{\theta_I})] \tag{1}
$$

This leads to an inequality of the form

$$
x\theta_S{}^{'} + y\theta_I{}^{'} + z\theta_S{}^{'}\theta_I{}^{'} < x\overline{\theta_S} + y\overline{\theta_I} + z\overline{\theta_S\theta_I} \tag{2}
$$

where,

$$x = [(B - A)(1 - H)(1 + \Delta\phi_S) - \Delta\phi_I F] \tag{3a}$$

$$y = [(D(E + F) + C(1 - H) + A(H - J))(1 + \Delta\phi_S) + (J - H) + \Delta\phi_I(G - E)] \tag{3b}$$

$$z = [((B - A)(H - J) - DF)(1 + \Delta\phi_S)]. \tag{3c}$$

Solving for the ESS is therefore merely a task of maximizing f(θ_S, θ_I) = xθ_S + yθ_I + z$\theta_S\theta_I$ for $0 \leq (\theta_S, \theta_I) \leq 1$. The results of this is shown in the main text.