

A Supplementary Appendix: Derivations

This appendix contains the derivations of many of the results in this paper, especially for sensitivities. Taking advantage of the freedom from length limits, I have tried to show the derivations step-by-step. Recall the definitions of the Hadamard product

$$\mathbf{A} \circ \mathbf{B} = (a_{ij} b_{ij}), \quad (\text{A-1})$$

the Kronecker product

$$\mathbf{A} \otimes \mathbf{B} = (a_{ij} \mathbf{B}), \quad (\text{A-2})$$

the vec operator

$$\text{vec} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a \\ c \\ b \\ d \end{pmatrix}, \quad (\text{A-3})$$

and Roth's theorem

$$\text{vec}(\mathbf{ABC}) = (\mathbf{C}^\top \otimes \mathbf{A}) \text{vec } \mathbf{B}. \quad (\text{A-4})$$

A.1 Variance in number of visits to transient states

The number of visits to transient state i , starting from transient state j , is ν_{ij} . The matrix of variances of the ν_{ij} is

$$\mathbf{V} = (V(\nu_{ij})) = (2\mathbf{N}_{\text{diag}} - \mathbf{I}_s) \mathbf{N} - \mathbf{N} \circ \mathbf{N} \quad (\text{A-5})$$

(Caswell 2006, derived from Theorem 3.1 of Iosifescu 1980) where \mathbf{N}_{diag} is a matrix with the diagonal elements of \mathbf{N} on its diagonal and zeros elsewhere; it can be written

$$\mathbf{N}_{\text{diag}} = \mathbf{I}_s \circ \mathbf{N} \quad (\text{A-6})$$

Differentiating both sides of (A-5) gives

$$\begin{aligned} d\mathbf{V} = & 2(\mathbf{I}_s \circ d\mathbf{N})\mathbf{N} + 2(\mathbf{I}_s \circ \mathbf{N})(d\mathbf{N}) - d\mathbf{N} \\ & - (d\mathbf{N}) \circ \mathbf{N} - \mathbf{N} \circ (d\mathbf{N}) \end{aligned} \quad (\text{A-7})$$

The next step is to apply the vec operator to both sides. The vec of a Hadamard product can be written in two ways:

$$\text{vec}(\mathbf{A} \circ \mathbf{B}) = \text{diag}(\text{vec } \mathbf{A}) \text{vec } \mathbf{B} = \text{diag}(\text{vec } \mathbf{B}) \text{vec } \mathbf{A}. \quad (\text{A-8})$$

Using this result and Roth's theorem (20) gives

$$\begin{aligned} d\text{vec } \mathbf{V} = & 2(\mathbf{N}^\top \otimes \mathbf{I}_s) \text{diag}(\text{vec } \mathbf{I}_s) d\text{vec } \mathbf{N} + 2 \left[\mathbf{I}_s \otimes (\mathbf{I} \circ \mathbf{N}) \right] d\text{vec } \mathbf{N} \\ & - d\text{vec } \mathbf{N} - 2\text{diag}(\text{vec } \mathbf{N}) d\text{vec } \mathbf{N} \end{aligned} \quad (\text{A-9})$$

Factoring out $d\text{vec } \mathbf{N}$ and using the chain rule gives the final result

$$\frac{d\text{vec } \mathbf{V}}{d\boldsymbol{\theta}^\top} = \left[2(\mathbf{N}^\top \otimes \mathbf{I}_s) \text{diag}(\text{vec } \mathbf{I}_s) + 2(\mathbf{I}_s \otimes \mathbf{N}_{\text{diag}}) \right]$$

$$-\mathbf{I}_{s^2} - 2\text{diag}(\text{vec } \mathbf{N}) \left[\frac{d\text{vec } \mathbf{N}}{d\boldsymbol{\theta}^\top} \right] \quad (\text{A-10})$$

A.2 Life expectancy

Let η_i be the time to absorbtion (i.e., death) of an individual currently in stage i . The vector $E(\boldsymbol{\eta})$ of expected values of the η_i is

$$E(\boldsymbol{\eta})^\top = \mathbf{e}^\top \mathbf{N} \quad (\text{A-11})$$

where \mathbf{e} is a vector of ones. Differentiating both sides gives

$$dE(\boldsymbol{\eta})^\top = \mathbf{e}^\top (d\mathbf{N}) \quad (\text{A-12})$$

Applying the vec operator gives

$$dE(\boldsymbol{\eta}) = (\mathbf{I}_s \otimes \mathbf{e}^\top) d\text{vec } \mathbf{N} \quad (\text{A-13})$$

Applying the identification theorem and the chain rule, and using (30) for the sensitivity of the fundamental matrix, gives

$$\frac{dE(\boldsymbol{\eta})}{d\boldsymbol{\theta}^\top} = (\mathbf{I}_s^\top \otimes \mathbf{e}^\top) (\mathbf{N}^\top \otimes \mathbf{N}) \frac{d\text{vec } \mathbf{U}}{d\boldsymbol{\theta}^\top} \quad (\text{A-14})$$

This gives the derivative of the entire vector of life expectancies. Suppose that stage 1 corresponds to birth. The life expectancy at birth is then

$$E(\eta_1) = \mathbf{e}^\top \mathbf{N} \mathbf{e}_1 \quad (\text{A-15})$$

where \mathbf{e}_1 is a vector with 1 in the first position and zeros elsewhere. Following the same derivation gives

$$\frac{dE(\eta_1)}{d\boldsymbol{\theta}^\top} = (\mathbf{e}_1^\top \otimes \mathbf{e}^\top) (\mathbf{N}^\top \otimes \mathbf{N}) \frac{d\text{vec } \mathbf{U}}{d\boldsymbol{\theta}^\top} \quad (\text{A-16})$$

A.3 Variance in longevity

The variance of the time to absorbtion satisfies

$$V(\boldsymbol{\eta})^\top = \mathbf{e}^\top \mathbf{N} (2\mathbf{N} - \mathbf{I}) - E(\boldsymbol{\eta}^\top) \circ E(\boldsymbol{\eta}^\top) \quad (\text{A-17})$$

(Caswell 2006, derived from Iosifescu (1980, Theorem 3.2)). Differentiating gives

$$dV(\boldsymbol{\eta})^\top = 2\mathbf{e}^\top (d\mathbf{N}) \mathbf{N} + 2\mathbf{e}^\top \mathbf{N} (d\mathbf{N}) - \mathbf{e}^\top (d\mathbf{N}) - 2E(\boldsymbol{\eta}^\top) \circ dE(\boldsymbol{\eta}^\top) \quad (\text{A-18})$$

Applying the vec operator and Roth's theorem (A-4), using (A-8) for the vec of the Hadamard product, gives

$$\begin{aligned} dV(\boldsymbol{\eta}) &= \left[2(\mathbf{N}^\top \otimes \mathbf{e}^\top) + 2(\mathbf{I}_s \otimes \mathbf{e}^\top \mathbf{N}) \right. \\ &\quad \left. - (\mathbf{I}_s \otimes \mathbf{e}^\top) \right] d\text{vec } \mathbf{N} - 2\text{diag}(E(\boldsymbol{\eta})) dE(\boldsymbol{\eta}) \end{aligned} \quad (\text{A-19})$$

Substituting (A-13) for $dE(\boldsymbol{\eta})$ gives

$$dV(\boldsymbol{\eta}) = \left[2(\mathbf{N}^\top \otimes \mathbf{e}^\top) + 2(\mathbf{I}_s \otimes \mathbf{e}^\top \mathbf{N}) \right.$$

$$- (\mathbf{I}_s \otimes \mathbf{e}^\top) - 2\text{diag}(E(\boldsymbol{\eta})) (\mathbf{I}_s \otimes \mathbf{e}^\top) \Big] d\text{vec} \mathbf{N} \quad (\text{A-20})$$

Using (30) for the sensitivity of \mathbf{N} , the identification theorem, and the chain rule finally leads to

$$\begin{aligned} \frac{dV(\boldsymbol{\eta})}{d\boldsymbol{\theta}^\top} &= \left[2(\mathbf{N}^\top \otimes \mathbf{e}^\top) + 2(\mathbf{I}_s \otimes \mathbf{e}^\top \mathbf{N}) \right. \\ &\quad \left. - (\mathbf{I}_s \otimes \mathbf{e}^\top) - 2\text{diag}(E(\boldsymbol{\eta})) (\mathbf{I}_s \otimes \mathbf{e}^\top) \right] (\mathbf{N}^\top \otimes \mathbf{N}) \frac{d\text{vec} \mathbf{U}}{d\boldsymbol{\theta}^\top} \end{aligned} \quad (\text{A-21})$$

A.4 Net reproductive rate

The net reproductive rate R_0 is given by the dominant eigenvalue of \mathbf{FN} . Let \mathbf{y} and \mathbf{x} be the right and left eigenvectors, respectively, of \mathbf{FN} , corresponding to R_0 . The matrix calculus version of the standard eigenvalue perturbation result (e.g., Caswell 1978) gives

$$\begin{aligned} dR_0 &= \mathbf{x}^\top d(\mathbf{FN}) \mathbf{y} \\ &= \mathbf{x}^\top [(d\mathbf{F})\mathbf{N} + \mathbf{F}(d\mathbf{N})] \mathbf{y} \end{aligned} \quad (\text{A-22})$$

Applying the vec operator to both sides gives

$$dR_0 = (\mathbf{y}^\top \mathbf{N}^\top \otimes \mathbf{x}^\top) d\text{vec} \mathbf{F} + (\mathbf{y}^\top \otimes \mathbf{x}^\top \mathbf{F}) d\text{vec} \mathbf{N} \quad (\text{A-23})$$

Applying the chain rule and the result (28) for $d\text{vec} \mathbf{N}$ gives the sensitivity of R_0 in terms of effects of the parameter vector $\boldsymbol{\theta}$ on the fertility matrix \mathbf{F} and the transient matrix \mathbf{U} :

$$\frac{dR_0}{d\boldsymbol{\theta}^\top} = (\mathbf{y}^\top \mathbf{N}^\top \otimes \mathbf{x}^\top) \frac{d\text{vec} \mathbf{F}}{d\boldsymbol{\theta}^\top} + (\mathbf{y}^\top \otimes \mathbf{x}^\top \mathbf{F}) (\mathbf{N}^\top \otimes \mathbf{N}) \frac{d\text{vec} \mathbf{U}}{d\boldsymbol{\theta}^\top} \quad (\text{A-24})$$

A.5 Cohort generation time

To derive the cohort generation time, we begin at time $t = 0$ with an individual newly born in stage j . This tiny cohort is described by an initial vector \mathbf{e}_j . The expected survivors of this cohort at time t are $\mathbf{U}^t \mathbf{e}_j$. The expected offspring produced by these survivors at time t are $\mathbf{FU}^t \mathbf{e}_j$. Adding these offspring over the lifetime of the cohort gives a vector of expected lifetime reproduction, of all types of offspring,

$$\begin{aligned} E(\text{total offspring}) &= \sum_{t=0}^{\infty} \mathbf{FU}^t \mathbf{e}_j \\ &= \mathbf{F} \left(\sum_{t=0}^{\infty} \mathbf{U}^t \right) \mathbf{e}_j \\ &= \mathbf{FN} \mathbf{e}_j \end{aligned} \quad (\text{A-25})$$

Let $\mathbf{m}^{(j)}(t)$ be the vector of offspring production at time t , expressed as a proportion of the lifetime total of the individual starting in stage j . Then

$$\mathbf{m}^{(j)}(t) = \text{diag}(\mathbf{FN} \mathbf{e}_j)^{-1} (\mathbf{FU}^t \mathbf{e}_j) \quad (\text{A-26})$$

If no offspring of some stage, say stage i , are produced, then set $m_i^{(j)}(t) = 0$.

The cohort generation time $\mu^{(j)}$ is the expectation of the distribution defined by $\mathbf{m}^{(j)}(t)$:

$$\begin{aligned}
\mu^{(j)} &= \sum_{x=0}^{\infty} x \mathbf{m}^{(j)}(x) \\
&= \sum_x \text{diag} (\mathbf{F} \mathbf{N} \mathbf{e}_j)^{-1} x \mathbf{F} \mathbf{U}^x \mathbf{e}_j \\
&= \text{diag} (\mathbf{F} \mathbf{N} \mathbf{e}_j)^{-1} \mathbf{F} \left(\sum_x x \mathbf{U}^x \right) \mathbf{e}_j
\end{aligned} \tag{A-27}$$

The summation can be simplified

$$\begin{aligned}
\sum_x x \mathbf{U}^x &= \mathbf{0} + \mathbf{U} + 2\mathbf{U}^2 + 3\mathbf{U}^3 + \dots \\
&= \mathbf{U} [\mathbf{0} + \mathbf{U} + 2\mathbf{U}^2 + \dots + \mathbf{I} + \mathbf{U} + \mathbf{U}^2 + \dots] \\
&= \mathbf{U} \left[\mathbf{N} + \sum_x x \mathbf{U}^x \right]
\end{aligned} \tag{A-28}$$

Solving this gives

$$\sum_x x \mathbf{U}^x = \mathbf{N} \mathbf{U} \mathbf{N} \tag{A-29}$$

Putting all the pieces together gives the generation time

$$\mu^{(j)} = \text{diag} (\mathbf{F} \mathbf{N} \mathbf{e}_j)^{-1} \mathbf{F} \mathbf{N} \mathbf{U} \mathbf{N} \mathbf{e}_j \tag{A-30}$$

A.5.1 Sensitivity of generation time

To differentiate (A-30) may seem complicated. To make life easier, define some notation,

$$\mathbf{X} = \text{diag} (\mathbf{F} \mathbf{N} \mathbf{e}_j) \tag{A-31}$$

$$\mathbf{r} = \mathbf{F} \mathbf{N} \mathbf{U} \mathbf{N} \mathbf{e}_j \tag{A-32}$$

in terms of which (A-30)

$$\mu^{(j)} = \mathbf{X}^{-1} \mathbf{r} \tag{A-33}$$

Differentiating gives

$$d\mu^{(j)} = d(\mathbf{X}^{-1}) \mathbf{r} + \mathbf{X}^{-1} d\mathbf{r} \tag{A-34}$$

Applying the vec operator

$$d\mu^{(j)} = (\mathbf{r}^\top \otimes \mathbf{I}) d\text{vec} \mathbf{X}^{-1} + \mathbf{X}^{-1} d\text{vec} \mathbf{r} \tag{A-35}$$

The same steps that led to equation (28) for $d\text{vec} \mathbf{N}$, and noting that \mathbf{X} is symmetric, leads to

$$d\text{vec} \mathbf{X}^{-1} = -(\mathbf{X}^{-1} \otimes \mathbf{X}^{-1}) d\text{vec} \mathbf{X} \tag{A-36}$$

The differential of $\text{vec } \mathbf{X}$ is obtained by writing

$$\mathbf{X} = \mathbf{I} \circ (\mathbf{F} \mathbf{N} \mathbf{e}_j \mathbf{e}^\top) \quad (\text{A-37})$$

Differentiating and using the rule (A-8) for the vec of a Hadamard product gives

$$d\text{vec } \mathbf{X} = \text{diag}(\text{vec } \mathbf{I}) [(\mathbf{e} \mathbf{e}_j^\top \mathbf{N}^\top \otimes \mathbf{I}) d\text{vec } \mathbf{F} + (\mathbf{e} \mathbf{e}_j \otimes \mathbf{F}) d\text{vec } \mathbf{N}] \quad (\text{A-38})$$

Differentiating \mathbf{r} and applying the vec operator gives

$$\begin{aligned} d\text{vec } \mathbf{r} &= [(\mathbf{N} \mathbf{U} \mathbf{N} \mathbf{e}_j)^\top \otimes \mathbf{I}] d\text{vec } \mathbf{F} + [(\mathbf{U} \mathbf{N} \mathbf{e}_j)^\top \otimes \mathbf{F}] d\text{vec } \mathbf{N} \\ &\quad + [(\mathbf{N} \mathbf{e}_j)^\top \otimes \mathbf{F} \mathbf{N}] d\text{vec } \mathbf{U} + [\mathbf{e}_j^\top \otimes \mathbf{f} \mathbf{N} \mathbf{U}] d\text{vec } \mathbf{N} \end{aligned} \quad (\text{A-39})$$

Whew!

Finally, substituting (A-36), (A-38) and (A-39) into (A-34), we obtain

$$\begin{aligned} \frac{d\boldsymbol{\mu}^{(j)}}{d\boldsymbol{\theta}^\top} &= -(\mathbf{r}^\top \otimes \mathbf{I}) (\mathbf{X}^{-1} \otimes \mathbf{X}^{-1}) \text{diag}(\text{vec } \mathbf{I}) \\ &\quad \times \left[(\mathbf{e} \mathbf{e}_j^\top \mathbf{N}^\top \otimes \mathbf{I}) \frac{d\text{vec } \mathbf{F}}{d\boldsymbol{\theta}^\top} + (\mathbf{e} \mathbf{e}_j \otimes \mathbf{F}) \frac{d\text{vec } \mathbf{N}}{d\boldsymbol{\theta}^\top} \right] \\ &\quad + \left\{ [(\mathbf{N} \mathbf{U} \mathbf{N} \mathbf{e}_j)^\top \otimes \mathbf{I}] \frac{d\text{vec } \mathbf{F}}{d\boldsymbol{\theta}^\top} + [(\mathbf{U} \mathbf{N} \mathbf{e}_j)^\top \otimes \mathbf{F}] \frac{d\text{vec } \mathbf{N}}{d\boldsymbol{\theta}^\top} \right. \\ &\quad \left. + [(\mathbf{N} \mathbf{e}_j)^\top \otimes \mathbf{F} \mathbf{N}] \frac{d\text{vec } \mathbf{U}}{d\boldsymbol{\theta}^\top} + [\mathbf{e}_j^\top \otimes \mathbf{F} \mathbf{N} \mathbf{U}] \frac{d\text{vec } \mathbf{N}}{d\boldsymbol{\theta}^\top} \right\} \end{aligned} \quad (\text{A-40})$$

This may be an impressive formula, but it is straightforward to compute, given the derivatives of \mathbf{U} , \mathbf{F} , and \mathbf{N} with respect to $\boldsymbol{\theta}$.

B Supplementary Appendix: Matrices for the time-varying example (*Lomatium bradshawii*)

The analysis of the stochastic model for *Lomatium bradshawii*, with $s = 6$ stages and $q = 4$ environments, leads to a fundamental matrix $\tilde{\mathbf{N}}$ of dimension 24×24 . In this section, I give the numerical results for the matrices that lead to, and are derived from, this fundamental matrix. These include:

1. The block diagonal matrix \mathbb{U} containing the transient matrices \mathbf{U}_i in each of the 4 environmental states.
2. The block diagonal matrix \mathbb{D} containing the transition matrix \mathbf{D} for the states of the environment.
3. The $sq \times sq$ matrix $\tilde{\mathbf{U}}$ giving the transitions for the time-varying Markov chain.
4. The fundamental matrix $\tilde{\mathbf{N}} = (\mathbf{I}_{sq} - \tilde{\mathbf{U}})^{-1}$
5. The matrices $\tilde{\mathbf{N}}^\ddagger$, $\tilde{\mathbf{N}}^\S$, and $\tilde{\mathbf{N}}^\heartsuit$, calculated from equations (82), (84), and (86), respectively.
6. The vector $E(\tilde{\boldsymbol{\eta}}|\epsilon_0)$ giving the life expectancy as a function of stage and initial environment, calculated from (87).
7. The vector $E(\tilde{\boldsymbol{\eta}}^\heartsuit)$ giving life expectancy as a function of stage, averaged over the stationary distribution of environments, calculated from (88).
8. The vector of conditional standard deviations of longevity as a function of stage and initial environment, calculated from (89).
9. The vector of unconditional standard deviations of longevity, including both within-environment and between-environment variance, calculated from (91).

		(B-3)																						
\tilde{U}		0.00	0.00	0.01	0.04	0.00	0.00	0.00	0.08	0.00	0.00	0.21	0.35	0.00	0.00	0.21	0.18	0.00	0.00	0.14	0.08	0.00	0.00	0.00
0.02	0.00	0.00	0.00	0.05	0.01	0.00	0.00	0.21	0.04	0.02	0.04	0.23	0.05	0.04	0.06	0.10	0.05	0.04	0.09	0.00	0.04	0.00	0.00	
0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
0.01	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.22	0.00	0.00	0.31	0.00	0.00	0.00	0.29	0.00	0.00	0.00	0.21	0.00	0.00	0.00	
0.00	0.00	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.10	0.23	0.00	0.00	0.21	0.36	0.00	0.00	0.23	0.53	0.00	0.00	0.00	0.00	0.00	
0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.14	0.03	0.00	0.00	0.32	0.06	0.01	0.03	0.53	0.00	0.06	0.05	0.04	0.34	0.04	0.08	
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.09	0.00	0.00	0.00	0.06	0.00	0.00	0.00	
0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.32	0.00	0.00	0.05	0.00	0.00	0.36	0.00	0.00	0.00	0.00	
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.05	0.15	0.00	0.00	0.00	0.26	0.20	0.00	0.00	0.43	
0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.05	0.00	0.00	0.01	0.17	0.06	0.00	0.05	
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	
0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.32	0.00	0.00	0.00	
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.28	

$\tilde{\mathbf{N}} =$	0.07	0.05	0.09	0.12	0.13	0.14	0.07	0.07	0.23	0.22	0.41	0.44	1.92	0.26	0.52	0.68	0.79	0.27	0.53	0.70	0.80	0.29	0.48	0.47	0.69
\mathbf{B}^{-1}	0.22	0.05	0.06	0.06	0.05	0.13	0.05	0.10	0.82	0.38	0.22	0.46	1.91	0.48	0.40	0.58	0.76	0.49	0.53	0.74	0.73	0.48	0.45	0.75	
\mathbf{C}	0.03	0.01	0.01	0.01	0.07	0.02	0.01	0.01	0.13	0.04	0.02	0.05	0.14	1.05	0.40	0.06	0.12	0.05	0.05	0.07	0.11	0.05	0.05	0.08	
\mathbf{D}	0.04	0.02	0.01	0.01	0.08	0.05	0.01	0.02	0.15	0.27	0.03	0.06	0.06	1.05	0.38	0.07	0.18	0.35	0.07	0.09	0.19	0.28	0.07	0.11	
\mathbf{E}	0.09	0.06	0.07	0.10	0.19	0.16	0.13	0.31	0.56	0.44	0.99	0.37	0.75	0.86	2.36	0.40	0.78	1.19	1.80	0.41	0.76	0.87	1.51		
\mathbf{F}	0.31	0.07	0.08	0.08	0.67	0.20	0.07	0.15	1.33	0.30	0.64	1.82	0.78	0.56	0.83	3.20	0.82	0.82	1.08	1.99	0.85	0.81	1.44		
\mathbf{G}	0.05	0.01	0.01	0.11	0.03	0.01	0.02	0.23	0.09	0.05	0.10	0.31	0.12	0.09	0.13	0.38	1.12	0.12	0.16	0.34	0.13	0.12	0.21		
\mathbf{H}	0.04	0.01	0.01	0.09	0.05	0.01	0.02	0.16	0.25	0.03	0.07	0.19	0.39	0.06	0.08	0.21	0.43	1.08	0.10	0.21	0.29	0.07	0.12		
\mathbf{I}	0.04	0.02	0.02	0.03	0.08	0.06	0.01	0.04	0.14	0.23	0.12	0.30	0.17	0.33	0.29	0.54	0.18	0.36	0.63	1.74	0.19	0.47	0.78	1.41	
\mathbf{J}	0.04	0.01	0.01	0.09	0.02	0.01	0.02	0.11	0.05	0.03	0.06	0.17	0.07	0.05	0.08	0.22	0.08	0.07	0.11	1.36	0.14	0.08	0.21		
\mathbf{K}	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.02	0.01	0.00	0.01	0.03	0.01	0.01	0.04	0.01	0.01	0.02	0.06	1.01	0.01	0.02			
\mathbf{L}	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.02	0.01	0.01	0.03	0.01	0.01	0.02	0.04	0.10	0.01	0.03	0.04	1.48	

(B-5)

$$\tilde{\mathbf{N}}^\ddagger = \left(\begin{array}{cccc|cccc|cccc|cccc|cccc} 1.00 & 1.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.76 & 0.33 & 0.25 & 1.98 & 1.62 & 1.50 & 1.63 & 0.50 & 0.39 & 0.45 & 0.48 & 0.45 & 0.43 & 0.50 & 0.44 & 0.47 & 0.32 & 0.44 & 0.00 \\ 0.45 & 0.20 & 0.18 & 0.21 & 0.94 & 0.49 & 0.14 & 0.40 & 2.38 & 2.06 & 1.72 & 2.47 & 1.32 & 1.14 & 1.14 & 1.38 & 1.23 & 1.13 & 0.38 \\ 0.38 & 0.14 & 0.14 & 0.18 & 0.79 & 0.36 & 0.10 & 0.26 & 1.41 & 1.25 & 0.72 & 1.56 & 2.60 & 2.66 & 2.35 & 3.06 & 1.46 & 1.68 & 1.74 \\ 0.45 & 0.11 & 0.13 & 0.14 & 0.96 & 0.35 & 0.10 & 0.23 & 1.85 & 1.16 & 0.50 & 1.10 & 2.49 & 1.62 & 0.99 & 1.57 & 3.97 & 2.74 & 2.73 \\ 0.05 & 0.01 & 0.01 & 0.01 & 0.11 & 0.04 & 0.01 & 0.02 & 0.15 & 0.08 & 0.04 & 0.09 & 0.22 & 0.10 & 0.08 & 0.12 & 0.29 & 0.14 & 1.46 \\ \end{array} \right)_{(B-5)}$$

$$\tilde{\mathbf{N}}^{\$} = \left(\begin{array}{ccccccc} 0.50 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.07 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.06 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.36 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ \hline 0.33 & 0.95 & 0.24 & 0.23 & 0.23 & 0.25 \\ 0.05 & 0.15 & 0.04 & 0.04 & 0.04 & 0.04 \\ 0.03 & 0.11 & 0.02 & 0.02 & 0.02 & 0.02 \\ 0.11 & 0.60 & 0.18 & 0.14 & 0.14 & 0.19 \\ \hline 0.18 & 0.39 & 1.18 & 0.65 & 0.65 & 0.88 \\ 0.03 & 0.06 & 0.18 & 0.10 & 0.10 & 0.14 \\ 0.02 & 0.05 & 0.13 & 0.06 & 0.07 & 0.07 \\ 0.09 & 0.17 & 0.86 & 0.50 & 0.51 & 0.46 \\ \hline 0.14 & 0.27 & 0.62 & 1.23 & 0.72 & 0.70 \\ 0.02 & 0.04 & 0.09 & 0.17 & 0.09 & 0.09 \\ 0.03 & 0.05 & 0.12 & 0.21 & 0.15 & 0.16 \\ 0.09 & 0.16 & 0.58 & 1.15 & 0.98 & 0.87 \\ \hline 0.20 & 0.41 & 0.96 & 1.30 & 2.11 & 1.63 \\ 0.03 & 0.07 & 0.16 & 0.22 & 0.34 & 0.26 \\ 0.03 & 0.06 & 0.12 & 0.16 & 0.24 & 0.18 \\ 0.03 & 0.06 & 0.20 & 0.32 & 0.79 & 0.69 \\ \hline 0.02 & 0.05 & 0.08 & 0.12 & 0.16 & 0.77 \\ 0.00 & 0.01 & 0.01 & 0.02 & 0.03 & 0.12 \\ 0.00 & 0.01 & 0.01 & 0.01 & 0.02 & 0.11 \\ 0.00 & 0.00 & 0.01 & 0.02 & 0.04 & 0.54 \end{array} \right) \quad (B-6)$$

$$\tilde{\mathbf{N}}^{\heartsuit} = \left(\begin{array}{ccccccc} 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.53 & 1.80 & 0.48 & 0.43 & 0.43 & 0.49 \\ 0.32 & 0.66 & 2.35 & 1.31 & 1.32 & 1.54 \\ 0.28 & 0.52 & 1.41 & 2.75 & 1.95 & 1.81 \\ 0.29 & 0.60 & 1.44 & 2.00 & 3.47 & 2.76 \\ 0.03 & 0.07 & 0.12 & 0.17 & 0.25 & 1.54 \end{array} \right) \quad (B-7)$$

$$\tilde{\boldsymbol{\eta}} = \begin{pmatrix} 3.08 \\ 1.79 \\ 1.96 \\ 1.78 \\ \hline 4.78 \\ 2.86 \\ 1.86 \\ 2.54 \\ \hline 6.28 \\ 4.94 \\ 3.43 \\ 5.70 \\ \hline 7.13 \\ 5.96 \\ 5.10 \\ 6.46 \\ \hline 7.42 \\ 6.13 \\ 6.19 \\ 7.91 \\ \hline 7.85 \\ 6.41 \\ 6.36 \\ 9.23 \end{pmatrix} \quad (\text{B-8})$$

$$\tilde{\boldsymbol{\eta}}^\heartsuit = \begin{pmatrix} 2.44 \\ 3.64 \\ 5.79 \\ 6.67 \\ 7.42 \\ 8.15 \end{pmatrix} \quad (\text{B-9})$$

$$\sqrt{V[\tilde{\boldsymbol{\eta}}]} = \begin{pmatrix} 4.19 \\ 2.46 \\ 2.68 \\ 2.66 \\ \hline 5.22 \\ 3.68 \\ 2.48 \\ 3.43 \\ \hline 5.89 \\ 5.31 \\ 4.51 \\ \frac{5.63}{6.01} \\ 5.65 \\ 5.43 \\ 5.96 \\ \hline 6.07 \\ 5.71 \\ 5.96 \\ 6.02 \\ \hline 5.97 \\ 5.77 \\ 5.63 \\ 6.07 \end{pmatrix} \quad (\text{B-10})$$

$$\sqrt{V[\tilde{\boldsymbol{\eta}}^\heartsuit]} = \begin{pmatrix} 3.56 \\ 4.54 \\ 5.72 \\ 5.96 \\ 6.04 \\ 6.05 \end{pmatrix} \quad (\text{B-11})$$