Supplementary results and analysis for "Infection state can affect host migratory decisions"

A Complete analytical expression of ESS

As stated in the main text, the matrix $\chi$ relating the number of susceptible and infected individuals from one generation to the next is defined as

$$
\begin{bmatrix}
S \\
I
\end{bmatrix}_{\tau+1} = \begin{bmatrix}
(1 + \Delta \phi_S)[(A(1 - \theta_S) + B\theta_S + C\theta_I)] + \Delta \phi_I(G\theta_I + E(1 - \theta_I) + F(1 - \theta_S)) + \Delta \phi_I(J\theta_I + H(1 - \theta_I)) \\
G\theta_I + E(1 - \theta_I) + F(1 - \theta_S)
\end{bmatrix} \begin{bmatrix}
S \\
I
\end{bmatrix}_\tau
$$
where

\[ A = \sigma (1 - c_R) e^{-\beta (T_1 + T_2)} \]
\[ B = \sigma (1 - c_M) e^{-\beta T_1} \]
\[ C = \sigma (1 - c_M) (1 - e^{-\beta T_1}) (1 - e^{-\gamma T_2}) \]
\[ D = \sigma (1 - c_M) (1 - e^{-\gamma T_2}) \]
\[ E = \sigma (1 - c_I) (1 - e^{-\beta T_1}) \]
\[ F = \sigma (1 - c_I) (1 - e^{-\beta T_2}) e^{-\beta T_1} \]
\[ G = \sigma (1 - c_M) (1 - c_I) (1 - e^{-\beta T_1}) e^{-\gamma T_2} \]
\[ H = \sigma (1 - c_I) \]
\[ I = \sigma (1 - c_M) (1 - c_I) e^{-\gamma T_2}. \]

We then use the Jury criteria for stability to determine the the value of \((\theta_S, \theta_I)\) at equilibrium. We find that the Jury criterion Tr[\(\chi(\theta_S', \theta_I')\)] - Det[\(\chi(\theta_S', \theta_I')\)] < 1 is the most sensitive indicator of the system’s stability and if it is not violated, then none of the other criteria are violated as well. Additionally for a stable system, the dominant eigenvalue of the system \((\lambda_1 \text{ henceforth})\) is equal to 1. For such a system, we can show that Tr[\(\chi((\theta_S, \theta_I))\)]-Det[\(\chi((\theta_S, \theta_I))\)] = 1. Therefore, we can say that a migration strategy \((\theta_S, \theta_I)\) is evolutionarily stable to invasions by a mutant with any other strategy \((\theta_S', \theta_I')\) if

\[ \text{Tr}[\chi(\theta_S', \theta_I')] - \text{Det}[\chi(\theta_S', \theta_I')] < \text{Tr}[\chi(\theta_S, \theta_I)] - \text{Det}[\chi(\theta_S, \theta_I)] \quad (1) \]

This leads to an inequality of the form

\[ x\theta_S' + y\theta_I' + z\theta_S' \theta_I' < x\theta_S + y\theta_I + z\theta_S \theta_I \quad (2) \]
where,

\[
x = [(B - A)(1 - H)(1 + \Delta \phi_S) - \Delta \phi_I F] \tag{3a}
\]

\[
y = [(D(E + F) + C(1 - H) + A(H - J))(1 + \Delta \phi_S) + (J - H) + \Delta \phi_I (G - E)] \tag{3b}
\]

\[
z =[((B - A)(H - J) - DF)(1 + \Delta \phi_S)]. \tag{3c}
\]

Solving for the ESS is therefore merely a task of maximizing \( f(\theta_S, \theta_I) = x\theta_S + y\theta_I + z\theta_S\theta_I \) for \( 0 \leq (\theta_S, \theta_I) \leq 1 \). The results of this is shown in the main text.