
Appendix 1–2
Appendix 1

Simulation codes of Figure 4

Supplied as a separate file: Appendix 1_Mathematica_code.nb

Appendix 2

Numerical analysis of non-structured one host–two partner module

Here, I provide numerical results to show that the partners are more likely to go extinct when the host has no stage structure. Suppose that a non-structured host \( H \) increases its population growth by exploiting two partners \( (P_1 \text{ and } P_2) \). By replacing the original notations \( H_J \) and \( H_A \) (for the juveniles and adults, respectively) by new one \( H \), the non-structure module can be derived follows:

\[
\frac{dH}{dt} = \left( r_H + \frac{b_{H,1}a_{H,1}P_1}{1+h_{H,1}a_{H,1}P_1} + \frac{b_{H,2}a_{H,2}P_2}{1+h_{H,2}a_{H,2}P_2} \right) H \\
- \frac{a_{1,H}H}{1+h_{1,H}a_{1,H}P_1} P_1 - \frac{a_{2,H}H}{1+h_{2,H}a_{2,H}P_2} P_2 - d_H H^2
\]

(A1a)

\[
\frac{dP_1}{dt} = \left( r_1 + \frac{b_{1,H}a_{1,H}H_J}{1+h_{1,H}a_{1,H}H_J} \right) P_1 - \frac{a_{H,1}P_1}{1+h_{H,1}a_{H,1}P_1} H - d_1 P_1^2
\]

(A1b)

\[
\frac{dP_2}{dt} = \left( r_2 + \frac{b_{2,H}a_{2,H}H}{1+h_{2,H}a_{2,H}H} \right) P_2 - \frac{a_{H,2}P_2}{1+h_{H,2}a_{H,2}P_2} H - d_2 P_2^2
\]

(A1c)

Note that except for the absence of the stage structure, all other assumptions regarding the basic model structure are the same as those made in the stage-structured community module in Eq. 1. The dynamics of this community module were numerically simulated in the same way as described in the main text. The results showed that the partners can go extinct irrespective of whether they and their host are facultative or obligate (the black regions). Notations and parameter conditions are the same as in Fig. 4.
Both partners are facultative

Partner 1 is facultative and partner 2 is obligate

Partner 1 is obligate and partner 2 is facultative

Both partners are obligate

Figure A1