Appendix 1

Data figure

Figure A1. Size variance in pike. Time series of the variance in body length (cm²) for offspring (age-1) pike from Windermere, UK, suggesting a negative trend over time (slope = −0.05; p-value = 0.08). Data were based on back-calculated lengths from individuals captured at age 3 and older. Colours indicate values above (blue, dark) and below (orange, light) the long-term average.
Appendix 2

Offspring survival

Figure A2. Cannibalism and background survival of offspring. Shown are offspring survival rates resulting from intercohort cannibalism (blue) and background mortality (orange) for low offspring size variance (top, $\sigma^2_{L1} = 3$) and high offspring size variance (bottom, $\sigma^2_{L1} = 10$) and moderate (left, $\beta_{cn} = 0.01$) and low (right, $\beta_{cn} = 0.001$) cannibalism intensity (500 time steps, all other parameters as in baseline model). Survival rates do not stabilize over time in case of low size variance and moderate cannibalism (top left) thus leading to unstable population dynamics.
Appendix 3
Incorporating competition into the model
Here we describe the methods and results for an extended version of our model that includes size-dependent intraspecific competition in addition to cannibalism. Size-dependent competition can be modelled in a number of different ways, although considering all possible effects of competition is beyond the scope of this study. We consider a scenario where size-dependent competition affects growth and survival and investigate how the main result (that offspring size variation promotes population stability) is affected.

In this model, competition is assumed to affect somatic growth and survival. Competitive interactions are governed by a competition kernel $C_{cp}(x, y)$, which describes the potential interaction strength between an individual of size $x$ and a competitor of size $y$ ($\int_0^{\infty} C_{cp}(x, y) dy = 1$). We assume that competition is most fierce between individuals of the same size, e.g. because they have similar diet or habitat preferences. Specifically, the competition intensity experienced by an individual of size $x$ due to potential competitors of size $y$ is given by a lognormal distribution with scale parameter $\sigma_{cp}$ and location parameter $\log(x) + \sigma_{cp}^2$. The competition kernel describes the potential effect of an individual of a given size on other individuals across the size range through competitive interactions (Fig. A3). Asymmetric competition for shared resources, for instance when juveniles and adults compete for the same prey species in the same habitat, is not reflected by this competition kernel.

Here, survival probability depends on background survival, competition, and cannibalism: $s_t(x) = s_b(x) s_{crt}(x) s_{cp,t}(x)$. The probability of surviving competition is given by $s_{cp,t}(x) = \exp(-\int_0^{\infty} \beta_{cps} y^{\alpha_{cps}} C_{cp}(x, y) n_t(y) dy)$, where $\beta_{cps}$ determines the effect size of competition on survival (higher $\beta_{cps}$ values imply stronger effects on survival), while $\alpha_{cps}$ scales the competitive intensity over competitor length $y$. Growth is also depending on competition in this model, through the mean length next year, given current length $x$ and population density $n_t(x)$. This is given by $\mu_{g,t}(x) = \ln[KL_\infty + (1 - K)x] - g_{cp,t}(x)$, where the last term describes the effect of competition and is scaled by the effect size $\beta_{cpG}$. $g_{cp,t}(x) = \beta_{cpG} x^{\alpha_{cpG}} \int_0^{\infty} n_t(x_{cp}) C_{cp}(x_{cp}, x) dx_{cp}$. The term $\alpha_{cpG}$ determines how the competitive intensity changes over length $x$, and integration is over competitor lengths $x_{cp}$.

Parameterization
The effect of competition on growth could not be estimated from the data and was thus varied in the model analysis. The default parameter was set to $\beta_{cpG} = 1 \times 10^{-5}$ (scaling parameter $\alpha_{cpG} = 1$) to
achieve reasonable effects of competition on the mean growth rate (i.e. biologically realistic ranges). Here, von Bertalanffy parameters were set to $K = 0.2$ and $L_{\infty} = 120$, so that growth patterns correspond to empirical patterns under intermediate population densities. The competition effect on survival was assumed to be an order of magnitude smaller than the effect of competition on growth ($\beta_{cpS} = 0.1 \beta_{cpG}$, and scaling parameter $\alpha_{cpS} = 0$).

Results

The shape of the population size distribution and the stability of the population dynamics also depend on the strength of competition, in addition to the strength of cannibalism. Increased competition causes a smoother size distribution with less distinct peaks for older cohorts, whereas cannibalism causes more pronounced peaks in the size distribution due to a reduction in density and thus competition (Fig. A4). Ultimately, very strong competition causes somatic growth to approach zero, which results in a unimodal size distribution. Previous work has shown that competition can lead to size convergence within cohorts due to exploitative interactions when small individuals are competitively superior, or increased size variation, for instance when alternative prey resources are available or social dominance structures allow for resource monopolization (Huss et al. 2007, 2008, 2010). The strengths of the intraspecific interactions thus determine if and where the transition from unstable to stable dynamics occurs (Fig. A5). As competition decreases and cannibalism increases, the transition appears and shifts to larger size variances. The degrees of competition and cannibalism were varied widely in order to cover a broad range of ecologically relevant interaction strengths. The lowest and highest values used for the two interactions thus represent large variation in growth rates and survival probabilities (Fig. A6). For example, for offspring of 25 cm (approximately the mean length at age-1 in the baseline model), the probability of surviving cannibalism varied between about 0.004 and 0.992, while the mean annual growth rate varied between 0% and 76% across all combinations of cannibalism and competition intensities considered in the analysis.
Figure A3. Illustration of the competition kernel. The left plot shows the size range of competitors as a function of the size of the focal individual (for different quantiles of the competition kernel, shaded grey areas). For three focal sizes (40, 80, 120 cm), the range (black lines) and mode (circles) of the kernel are highlighted. The right plot shows the realized variance of the competition kernel for all focal sizes (taking into account the lower and upper size limits) to illustrate that the width of the competition window reaches a maximum at intermediate sizes.
Figure A4. Stable size distribution for different degrees of competition and cannibalism. Thick black lines indicate the size distribution after the maximum number of time steps (t=1000). Thin grey lines indicate the size distribution 1, 3, 5, and 10 years earlier to indicate whether the dynamics are stable or unstable. The default value of 13 was used for the offspring size variance. Cannibalism intensity decreases from left ($\beta_{cn} = 0.1$) to right ($\beta_{cn} = 0.001$), and competition intensity increases from bottom ($\beta_{cpG} = 1e^{-6}$) to top ($\beta_{cpG} = 1e^{-4}$).
Figure A5. Bifurcation diagrams of population density against the variance in offspring length. Projections were run for 1000 time steps and population density was sampled for the last 100 time steps. Cannibalism intensity decreases from left ($\beta_{cn} = 0.1$) to right ($\beta_{cn} = 0.001$), and competition intensity increases from bottom ($\beta_{cpg} = 1e^{-6}$) to top ($\beta_{cpg} = 1e^{-4}$). These values were chosen to represent a broad range of survival probabilities and mean growth rates.
Figure A6. Cannibalism survival probabilities and mean growth rates for the range of cannibalism and competition intensities considered in the additional model analyses. The dotted line represents zero growth. The plot was produced by setting $\sigma_L^2$ to 16 instead of the default value, which resulted in stable dynamics for all tested combinations of cannibalism and competition intensities.
Appendix 4
Additional sensitivity tests

The following sensitivity analyses were performed to further scrutinize further our results with respect to important parameters in the model. First, a bifurcation analysis for different mean values of the offspring size distribution showed that the shift to unstable dynamics at low size variance occurred for a large range of mean sizes of about 21–27 cm, but that this pattern disappeared at smaller mean sizes (Fig. A7). This suggests that a transition to unstable dynamics at low size variance is expected for the range of mean offspring sizes observed in Windermere pike (~21–26 cm). Second, in addition to cannibalism intensity, the relative victim-to-cannibal size ratio and the width of the cannibalism kernel affect the size distribution and population stability. Population dynamics stabilize at a higher mean victim-to-cannibal size ratio (Fig. A8), and a wider cannibalism kernel (Fig. A9), because a wide diet range of cannibals contributes to a smoother size distribution through spreading the predation risk across the victim size range. Finally, we show that and the stability-instability transition occurs at lower variance values when individual growth variation is increased (Fig. A10).

![Figure A7. Bifurcation diagrams of population density over variance in offspring size for different values of mean offspring size. In order to investigate whether the shift to unstable dynamics at low size variance would occur at different values of mean offspring size (default value at ~23 cm), we fixed the mean offspring size such that it would not depend on total egg number. The range of mean offspring sizes observed in Windermere pike across years is about 21–26 cm, and the range shown here is 19 cm (left) to 27 cm (right).](image-url)
Figure A8. Bifurcation diagrams of population density against variance in offspring length for different values of the mean victim-to-cannibal size ratio ($\mu_{\text{cn}}$). Values of $\mu_{\text{cn}}$ range from −1.3 (left) to −1.7 (right). The location parameter $\mu_{\text{cn}}$ is defined on the log scale.

Figure A9. Bifurcation diagrams of population density against variance in offspring length for different values of the standard deviation in the victim-to-cannibal size ratio. Values of $\sigma_{\text{cn}}$ range from 0.2 (left) to 0.4 (right).

Figure A10. Bifurcation diagrams of population density against variance in offspring length for different values of growth variation ($\tau_g$). Values of $\tau_g$ range from 1 (left) to 9 (right).