Appendix 1. Extended methods and results

A1.1 Parameters and units

The half saturation constant (H) is expressed in grams biomass per unit volume, the unit volume can be changed by changing H. Biomass densities are expressed in gram biomass per the unit volume defined by H. The half saturation constant (H) and the metabolic rate (T) are set to 1 and the parameters r, a and m are expressed as multiples of T. We refer to de Roos et al. (2008a) for more details on the scaling and derivation of parameters and units. The only difference between the consumer-resource model in de Roos et al. (2008a) and our basic model is that here both stages have an explicit parameter scaling their competitiveness (here q_j and q_a are 0.5 or 1.0, compared to de Roos et al. where q_a is 0.5 or 2, and “q_j” implicitly is 1).

A 1.2 Unstructured model

For comparison we used an unstructured model which is equivalent to the Yodzis and Innes model (1992) that has been used as the basic unit for both food-web models assessing stability (McCann et al. 1998) and stage-structured biomass models used for investigating the consequences of biomass overcompensation (de Roos et al. 2007, 2008a). The consumer population is now denoted with C instead of J and A (Eq. A1). The parameters remain the same.
as for the stage-structured model, except for that the parameter $q_c$, describing the competitiveness of the consumer, now represents the whole population (varied $q_c$; 0.5/0.75/1).

$$\frac{dC}{dt} = \frac{c q a c r R}{H + R} C - \frac{a p C}{H + C} P - (m + T) C \quad (A1)$$

**A 1.3 Dynamics of the biomass model compared to a fully size-structured model**

The biomass model used here approximates the dynamics of a fully size-structured model, however, the dynamics are not always identical. When juveniles are superior competitors and juvenile-driven cohort cycles occur in the fully size-structured model, there are some parameter ranges within which dynamics are stable in the biomass model (de Roos et al. 2007). This means that our analysis potentially overestimates the stability of the system when juveniles are superior competitors compared to if a fully size-structured model had been used. Note however that there are no large qualitative differences between the results for models where juveniles or adults are superior competitors (Fig. 3-4 main paper). For example, we still see the refuge effect in both cases. Importantly, the biomass model captures the patterns in $\lambda_{max}$, even if $\lambda_{max}$ does not turn positive (i.e. unstable) at the same parameter value as for the size-structured model (when juveniles are superior competitors). Hence, the discrepancy between models should not affect the qualitative patterns, but rather the precise parameter range that produce different dynamics.

Due to the above discrepancies, we use the case when adults are superior for investigating selectivity (section A2.3). In our example with the small destabilization effect at weak interaction strength (boosting of cohort cycles, Fig. 4b main paper, Appendix A2.2, Fig. A4), adults are superior competitors, and hence the dynamics should be reliable (de Roos et al. 2007).

It is unlikely that the model discrepancy has any impact on the unstable phase when two predators are present; the dynamics are not similar to juvenile-driven cycles in this case (Fig. 5...
main paper, Fig. A5). If anything, we are overestimating stability by using the biomass model, so the destabilizing effect of combining two predators would be underestimated and not overstated.

Finally, the result that increasing consumer maximum consumption rate does not lead to consistent higher amplitude fluctuations (Fig. 2b, A1b) specifically relates to the nature of cohort cycles. Because of this, we confirmed that the same result was present in a fully size-structured model, using code from de Roos et al. (2007, not shown).

A 1.4 Full equations for when a predator is feeding on juvenile and adult consumers

\[
\frac{dR}{dt} = rR \left(1 - \frac{R}{K}\right) - \frac{qa_cr}{H+R} J - \frac{qa_c}{H+R} A \quad (A2)
\]

\[
\frac{dJ}{dt} = \left(\frac{cq_{a_c}R}{H+R} - T\right) J + \left(\frac{cq_{a_a}R}{H+R} - T\right) A - \left(\frac{cq_{a_c}R}{H+R} - T\right) \left(\frac{m^+ a_p^P}{H+J+A}\right) J - mJ - \frac{a_p^P}{H+J+A} J \quad (A3)
\]

\[
\frac{dA}{dt} = \left(\frac{cq_{a_c}R}{H+R} - T\right) J - mA - \frac{a_p A}{H+J+A} P \quad (A4)
\]

\[
\frac{dP}{dt} = \frac{c_p a_p l}{H+J+A} P + \frac{c_p a_p A}{H+J+A} P - (m_p + T_p) P \quad (A5)
\]
Appendix 1. Extended results

A 2.1 Consumer resource model for when adults are superior competitors

Here we give a detailed description of what happens when we increase the interaction strength in the consumer-resource model for the case when adults are more competitive (equivalent to Fig. 2a in the main article where juveniles are more competitive). The consumer enters the system when it attains a maximum consumption rate that permits a feasible solution (at $a_{ct} = 6.5$ in Fig. A1a). After becoming feasible, further increases in consumption rate during this phase increase the energy to the consumer raising the population away from dangerously low densities. Interestingly, this stabilizing phase corresponds to real dominant eigenvalues ($\lambda_{\text{max}}$), which means that populations grow monotonically back to the equilibrium after a small perturbation. In such a case of monotonic growth, it is not surprising that increases in growth rates of the consumer (more energy) allow it grow more rapidly back to the equilibrium increasing the stability (Fig. A1a; stabilizing phase).
At precisely the point where the eigenvalue become complex, the higher maximum consumption rate by the consumer drives a destabilizing phase. Here, and in a manner identical to classical consumer-resource theory (Murdoch et al. 2003, Rip and McCann 2011), increased energy flux through a heightened consumer maximum consumption rate promotes overshoot dynamics. Thus, despite a rapid return towards the equilibrium the populations begin to oscillate around the equilibrium, slowing down return times. This manifests as a phase that shows increasingly larger $\lambda_{\text{max}}$ (Fig. A1a; destabilizing phase). Ultimately, the interaction is destabilized (at approx. $a_{ct} = 7.4$ the interaction undergoes a Hopf bifurcation) producing oscillations that can be stage-structured adult-driven cohort cycles (Fig. A1b) or classical consumer-resource cycles (de Roos and Persson 2013b).

After the destabilization, the amplitude of the adult-driven cohort cycles increase as consumer maximum consumption rate is increased (Fig. A1b). However, as was the case for juvenile-driven dynamics (main paper Fig. 2 b), increasing the maximum consumption rate even further resulted in decreasing cycle amplitude (Fig. A1b).

**A 2.2 The small destabilizing effect from one predator: time-series simulations**

When a predator feeds on the adult stage and adult consumers are superior competitors, the predator initially seemed to have a destabilizing effect as it invaded (an increase in $\lambda_{\text{max}}$ seen in Fig. 4b inset). Since changes in $\lambda_{\text{max}}$ are not reliable when $\lambda_{\text{max}}$ is positive, we performed time-series simulations to investigate the effect of this invasion on the population dynamics. The amplitudes of the population cycles present increased as the predator entered (Fig. A2c-d), while the dynamics were similar before and after the invasion (Fig. A2a-b). The dynamics are characterized by juveniles and adults peaking out of phase, and the amplitudes of the cycles were small, which suggests that cohort cycles are at least a part of the dynamics. Typically, in 5
adult-driven cohort cycles, the ability of adults to temporarily depress the resource level is vital. Here the peaks in adult biomass were higher with a predator present (although the average biomass was lower, Fig. A2d).

Note that this is a small effect that occurred for a narrow parameter range in our example, and further investigation is needed to assess its importance, as well as the mechanism behind it.

Figure A2. Time-series simulations for cohort cycles with and without a predator present, adult consumers are superior competitors and the predator is feeding on adults, corresponding to inset in Fig. 4b main paper; (a), dynamics without a predator present (juvenile consumers in dashed black, adult consumers in full black), (b), dynamics with a predator feeding on adults present (juveniles in dashed black, adults in full black, predator in grey), (c), resource levels with (in black) and without predator (in grey), (d), adult consumer density with (in black) and without predator (in grey).
A 2.3 Variation in carrying capacity for the food chain case with one predator feeding on the adult stage: emergence of cycles

The food chain case with a predator feeding on adults while juvenile consumers are superior competitors is here examined in more detail. The bifurcation over predator maximum consumption (Fig. 4a main paper) resulted in a stability response with a 'hump in $\lambda_{\text{max}}$'. As carrying capacity of the system was increased (Fig. A3a), $\lambda_{\text{max}}$ became $>0$ and cycles emerged. The peak in predator biomass follows the peak in adult consumers with a lag of approximately 1/3 or 1/4 of the cycle length, which could be indicative of predator prey cycles. However, the juvenile and adult consumer are clearly out of phase (Fig. A3b), and the peaks in consumer biomasses are not very distinct (compared to for example the 2 predator case in Fig. A5), nor high. Also, the fact that the cycles disappear at higher predator max consumption rate, in spite of that the predator biomass increases (not shown), suggests that this is not predator-prey cycles. This rather points to that these are stage-structure driven cohort cycles. We leave further investigation of these cycles for future studies but note that the 'hump in $\lambda_{\text{max}}$', which enables these kind of dynamics, seems to be linked to biomass overcompensation in the juveniles. Also, note that we here have a case where not only increasing interaction strength can have a destabilizing effect on dynamics in a food chain, but that decreasing interaction strength can have a destabilizing effect (when decreasing maximum predator consumption from high values).
Figure A3. Additional results for the food chain case with a predator feeding on adult consumers, and juveniles are superior competitors; (a), Stability measures ($\lambda_{\text{max}}$) as a function of predator maximum consumption rate for different productivity levels, ($K = 1$ dotted line, $K = 2$ full, $K = 3$ dashed, $K = 4$ dot-dashed), (b), Time-series for a case with $\lambda_{\text{max}} > 0$ ($K=3$, $a_{pa}=75$), with biomass of the predator on adults (black full line), juvenile consumers (dashed), adult consumers (full grey) and resources (dotted line).

A 2.4 Stage structured model with predator feeding on both stages

To investigate the lack of an unstable phase at high maximum consumption rate we also allowed a predator to feed on both stages simultaneously (Fig. 1d main paper), and on an unstructured consumer population. We then looked at the effect of changing both predator maximum consumption rate and carrying capacity for predators with different selectivity (Fig. A4).

When the predator is feeding exclusively on the juvenile stage, increasing the predator’s maximum consumption rate does not result in destabilization for this parameter range (Fig. A4a, adults superior competitors, $K=2$ is as Fig. 3b in main paper). However, when the predator is feeding on both juvenile and adult consumers, the system is prone to turn unstable. The bifurcation over predator maximum consumption rate shows the classic checkmark in $\lambda_{\text{max}}$ for default values (Fig. A4b, for $K=2$), and is even less stable at higher levels of productivity.

In these two examples (Fig. A4a vs A4b) the maximum consumption rates are not directly comparable since a predator feeding on both stages can be expected to have a larger impact. However, even if the consumption rate with selective feeding (A3b) is increased to very high
levels compared to unselective feeding (A3a), the system does not turn unstable. We argue that there is a clear qualitative pattern in that selective feeding on one stage does not result in full destabilization (positive $\lambda_{max}$), while feeding on both stages does.

The model lacking stage structure displayed the classical destabilization with increasing maximum consumption rate (Fig. A4c). The consumer feeding efficiency was varied by changing the parameter $q_c$ (Eq. A1), this was done to facilitate a comparison with the stage-structured model. Since $q_j$ and $q_a$ (parameters affecting the feeding efficiency of the juvenile and adult stages in the stage-structured model) varies between 0.5 and 1, the examples where $q_c$ equals 0.5 and 1 (Fig. A4c) represent the extreme minimum and maximum feeding efficiencies in the unstructured model, while $q_c = 0.75$ is a somewhat arbitrary intermediate example. The transition from stable to unstable dynamics occurred around $a_p = 21$ for the highest $q_c$. (Fig. A4c). In comparison the transition occurred around $a_p = 10$ in the stage-structured model (Fig. A4b). Hence, here the unstructured model is more stable, in the sense that it needs a higher predator maximum consumption rate to turn unstable. However, the difference is minimal, and the comparison is not complete in any way. Investigating other parameter values also revealed richer patterns for the different models (we leave this, however, for future studies).

In conclusion, when the predator was feeding on both stages, the system was more prone to turn unstable compared to when the predator was selectively feeding on one stage, and was less stable than the unstructured model. Here we would like to emphasize that this is no way a complete comparison between unstructured and stage-structured models, but simply an example.
Figure A4. Stability measures ($\lambda_{\text{max}}$) as a function of predator maximum consumption rate; (a), for different productivity levels with a predator feeding on the juvenile consumer stage only (adults superior competitors, $K = 2$ full line, $K = 3$ dashed, $K = 4$ dotted), (b), for different productivity levels with a predator feeding on both consumer stages (adults superior competitors), (c), with a predator feeding on an unstructured consumer population (varied consumer feeding efficiency: $q_c = 1$ full line, $q_c = 0.75$ dashed, $q_c = 0.5$ dotted, while $K = 2$).
**A 2.5 Additional results for when two predators are present**

**Details for when a predator feeding on adults is invading and juvenile consumers are superior competitors:**

The details presented here are for the case when a **predator feeding on adults is invading a system** where a predator feeding on juveniles is already present (Fig. 5c-d and Fig. 6c-d main paper). When **juveniles are superior competitors** the resident predator does not help the predator feeding on adults to invade the system, it enters at the same maximum consumption rate as when the predator feeding on juveniles is absent (compare Fig. 5c with Fig. 4a, main paper). When the predator feeding on adults enters the system, it has a negative effect on the biomass of the resident predator (Fig. 6c) and the adult consumers (Fig. A5c). As the invader’s maximum consumption rate is increased, there is a destabilizing effect and the system turns unstable ($\lambda_{\text{max}}>0$) as shown in the main paper (Fig. 5c). Eventually the resident goes extinct ($a_{pa} \approx 1130$).

**Details for when a predator feeding on adults is invading and adult consumers are superior competitors:**

When **adult consumers are superior competitors**, biomass overcompensation takes place in the adult stage. Hence, in this case, the invasion of the predator feeding on adults is helped by the predator feeding on juveniles; it enters at lower maximum consumption rate compared to when the other predator is not present (i.e. emergent facilitation, compare Fig. 4b with Fig. 5d). The invasion has a negative effect on the resident predator biomass (Fig. 6d). The $\lambda_{\text{max}}$ goes to zero at the invasion, but overall there is a destabilizing of the system that we attribute to a direct transition to the phase where the combined effect of the two predators causes destabilization (as present in all cases in Fig. 5). At even higher maximum consumption rates, the predator feeding on juveniles goes extinct ($a_{pa} < 1200$).
Figure A5. Consumer biomass responses for bifurcations over predator maximum consumption rate when a predator is invading a system with a resident predator present, corresponding to figures 5 and 6 in the main paper. (a), for when the predator of juveniles is invading a system where the predator feeding on adults is the resident ($a_{pa}=50$) and juvenile consumers are superior competitors, (b), as for a but when adult consumers are superior ($a_{pa}=400$), (c), for when the predator of adults is invading a system where the predator feeding on juveniles is the resident ($a_{pj}=50$) and juvenile consumers are superior competitors, (d) as for c but when adult consumers are superior ($a_{pj}=20$). The adult consumer biomass is shown in dashed lines and the juvenile consumer biomass is shown in full lines.
**Time-series with two predators present:**

Here we present time-series simulations corresponding to the unstable phase occurring at intermediate interaction strength when two predators are present (Fig. A6, corresponding to Fig. 5 in the main paper). The fluctuations in the consumer stages are relatively synchronous, although the adults peak before the juveniles. Correspondingly, the predator feeding on adults peaks before the predator feeding on juveniles (Fig. A6). The predator feeding on adults is lagging behind the adult consumers with about \( \frac{1}{4} \) of the cycle period (or less), which indicates that we see a kind of predator-prey cycles. This is further supported because the pattern occurs irrespective of which consumer stage, juvenile or adult, is the superior competitor (Fig. A6).

Figure A6. Time-series simulations for the cases when two predators are present, examples corresponding to the unstable phases in Fig. 5 in the main paper, with (a) corresponding to Fig. 5a in the main paper \((a_{pj}=38 \text{ and } a_{pa}=50)\) where juvenile consumers are superior competitors, in (b) corresponding to Fig. 5b in the main paper \((a_{pj}=52 \text{ and } a_{pa}=300)\) where adult consumers are superior competitors, in (c) corresponding to Fig. 5c \((a_{pj}=50, a_{pa}=65)\) where juvenile consumers are superior competitors, in (d) corresponding to Fig. 5d \((a_{pj}=20, a_{pa}=130)\) where adult consumers are superior competitors. Biomass of the predator on adults (gray full line), the predator on juveniles (dashed gray), juvenile consumers (full black line) and adult consumer (black dotted) is shown.
Figure A6.
References

Appendix B. Additional model runs

Appendix B1. Additional results for the food chain case when juveniles are superior competitors.

We here perform bifurcations over the maximum consumption rate of a predator that feeds on adult consumers. The default parameters resulted in an interesting stability response (Fig. 4a main paper, Fig. B1b full line), with a double check-mark pattern in \( \lambda_{\text{max}} \). We explore this result further, first by varying the maximum consumption rate of the consumer (\( a_{\text{cr}} \)) and then the carrying capacity (\( K \)).

B1.1 Variations in maximum consumption rate of the consumer (\( a_{\text{cr}} \))

The 'hump' in \( \lambda_{\text{max}} \) occurs for a larger range of predator maximum consumption rate (\( a_{\text{pj}} \)) when consumer maximum consumption rate (\( a_{\text{cr}} \)) is increased (Fig. B1a, full line is default parameters). The hump overlaps with the range of biomass overcompensation in the juvenile stage (not shown). At even higher \( a_{\text{cr}} \), the biomass overcompensation eventually disappears and \( \lambda_{\text{max}} \) becomes closer to zero. However, the system does not turn unstable even at high \( a_{\text{cr}} \) (\( a_{\text{cr}} = 30 \), Fig. B1a). At very low \( a_{\text{cr}} \) the hump in \( \lambda_{\text{max}} \) also goes away (\( a_{\text{cr}} = 6.3 \), fig. B1b), while weak biomass overcompensation in the juvenile stage is still present (not shown).

Figure B1. Stability measures for bifurcations over predator maximum consumption rate (\( a_{\text{pa}} \)) when the maximum consumption rate of the consumer is varied; (a) stability measure (\( \lambda_{\text{max}} \)) with high consumer maximum consumption rate (default \( a_{\text{cr}} = 10 \) in full, \( a_{\text{cr}} = 12 \) in dashed, \( a_{\text{cr}} = 16 \) in dotted, \( a_{\text{cr}} = 30 \) in dot-dashed) (b) stability measure (\( \lambda_{\text{max}} \)) with low consumer maximum consumption rate (default \( a_{\text{cr}} = 10 \) in full lines, \( a_{\text{cr}} = 8 \) in dashed, \( a_{\text{cr}} = 6.3 \) in dotted). The predator is feeding on adult consumers and juvenile consumers are superior competitors.
Before the predator enters (far left a, b), some consumer population fluctuates ($\lambda_{\text{max}} > 0$) and the lines shown represent unstable equilibrium densities.

**B1.2 Variations in carrying capacity (K)**

With increasing carrying capacity the 'hump in $\lambda_{\text{max}}$' becomes higher. Eventually $\lambda_{\text{max}}$ becomes positive and cycles emerge (Fig. B2a, and also shown in Appendix A, Fig. A3a). The peak in predator biomass follows the peak in adult consumers with a lag of approximately 1/3 or ¼ of the cycle length, which could be indicative of a kind of predator prey cycles (Fig. B3, also shown in Appendix A; Fig A3b). However, the juvenile and adult consumer are clearly out of phase, the peaks in consumer biomasses are not very distinct. Also, the fact that the cycles disappear at higher predator max consumption rate, in spite of that the predator biomass increases, suggests that this is not the case. This rather points to that stage-structured driven cohort cycles are a part of the dynamics. We note that 'hump' in $\lambda_{\text{max}}$ (enabling these kind of dynamics) seems to be linked to biomass overcompensation in the juveniles (shown in Fig. B2b), or perhaps the transition of energy from the juvenile to the adult stage. Parts of this is also described in the main paper and Appendix A (for publication).
Figure B2. Results for bifurcations over predator maximum consumption rate ($a_{pa}$) when carrying capacity is varied, (a) stability measure ($\lambda_{max}$) with varied carrying capacity ($K=1$ in dotted, default $K=2$ in full, $K=3$ in dashed, $K=4$ in dot-dashed lines) (b) juvenile biomass with varied carrying capacity ($K=1$ in high dotted, $K=2$ in full, $K=3$ in dashed, $K=4$ in low dotted lines). The predator is feeding on adult consumers and juvenile consumers are superior competitors. Before the predator enters (far left a), some consumer population fluctuates ($\lambda_{max} >0$) and the lines shown represent unstable equilibrium densities.

Figure B3. Time-series dynamics for the case when the predator is feeding on adult consumers and juvenile consumers are superior competitors (default parameters and $K=4$, $a_{pa}=75$). Biomass of the predator on adults (gray full line, juvenile consumers (full black line) and adult consumer (black dotted line) is shown.
Appendix B2. Additional results for the 2 predator case, when adult consumers are superior competitors, and the predator feeding on juveniles is invading.

We perform bifurcations over the maximum consumption rate of the invading predator feeding on juveniles (and adult consumers are superior competitors, as in the main paper Fig. 5b). This result was especially interesting since it revealed that weak interactions can have destabilizing effects. We explore this further by varying the maximum consumption rate of the resident predator (feeding on adults).

Emergent facilitation and the weak destabilizing interaction are present over a wide range of resident predator maximum consumption rate (Fig. B4). When the resident is made very strong ($a_{pa} = 1200$, grey line Fig. B4a) the weak destabilizing effect is almost completely gone, we see only a slight increase in resident biomass as the resident enters (Fig. B4b, hardly visible in this format).

A second interesting result was the destabilizing effect that caused cycles to emerge at intermediate interaction strength (Fig. 5 main paper). When the resident maximum consumption rate is increased, there is a shift of the 'area of destabilization arising from the two predators combined' towards higher invader maximum consumption rate ($a_{pj}$). This
demonstrates that the unstable phase is not a consequence of “combined high consumption rates of the two predators”, but rather that the predation pressure from the different predator populations are relatively balanced.

Figure B4. Bifurcations over the invading predator’s maximum consumption rate \(a_{pj}\), when maximum consumption rate of the resident predator is varied, (a) stability measure \(\lambda_{\text{max}}\) with the maximum consumption rate of the resident predator is varied (default \(a_{pa} = 400\) in full black, \(a_{pa} = 500\) in dashed, \(a_{pa} = 600\) in dotted, \(a_{pa} = 800\) in dot-dashed and \(a_{pa} = 1200\) in full grey). (b) Biomass of the predator feeding on adults (full lines) and the predator feeding on juveniles (dashed line) for default (in grey) and high (in black) resident maximum consumption rate.

Appendix B3. Additional results for the 2 predator case when juveniles are superior competitors and a predator feeding on adults is invading. We again perform bifurcations over the maximum consumption rate of the invading predator (feeding on adults, as in the main paper Fig. 5b). Here the maximum consumption rate of the resident predator (feeding on juveniles) is varied.
Variation in resident predator maximum consumption rate ($a_{pj}$), did not reveal any novel patterns. We still see only one hump in $\lambda_{max}$, and no initial destabilizing effect (no positive effects on the resident predator biomass from the invasion), even if the resident predator is made weaker. Again, the 'hump in $\lambda_{max}$' is shifted towards higher invader maximum consumption rate as the resident is made stronger (increasing $a_{pj}$).

Figure B2. Stability measure ($\lambda_{max}$) for bifurcations over predator maximum consumption rate ($a_{pa}$) when the maximum consumption rate of the resident predator ($a_{pj}$) is varied ($a_{pj} = 25$ in dotted, default $a_{pj} = 50$ in full, and $a_{pj} = 70$ in dot-dashed lines).