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## Appendix 1

There may be confusion regarding the mechanism we describe, and that dubbed the “inflationary effect” first explored by Holt, Gonzalez and coauthors (2002, 2003). These authors also investigated the effects of autocorrelated environmental stochasticity on populations, but were concerned with the long-term persistence of metapopulations with bounded dynamics. Their conclusions and approach reflect this interest, but do not apply to the transient dynamics of isolated populations with unbounded growth that we examine. The key difference here is that we never find that autocorrelated variation can increase the persistence of isolated populations with growth rates below replacement.

Below we briefly describe the inflationary effect, and the analysis that points to its existence. We then contrast this result with the arguments that apply to short-term dynamics of isolated populations. We show that, unlike the long-term inflationary effect for metapopulations, positively autocorrelated noise may produce either positive or negative covariance between population growth rates and population density. In some cases, autocorrelated stochasticity may increase short term population density, but it does not increase long-term persistence as in the inflationary effect. Moreover, increases average population density are not necessarily correlated with establishment before extinction.

Gonzalez and Holt (2002), expanding on an argument from Levins and Puccia (1985) show that autocorrelated fluctuations can increase average population size in a sink population of a metapopulation, even though such fluctuations enhance extinction risk in closed populations. If the rate of change in population size is:  $\frac{dN}{dt} = f(t)N + I$ , where the population growth rate,  $f$ , fluctuates through time, and the immigration rate is given by  $I$ . If  $I=0$ , then  $N(T) = N(0)\left(\exp\langle f \rangle T\right)$ , where  $\langle f \rangle$  is the average rate of population growth over  $T$ , and of course, if  $\langle f \rangle$  is negative as  $T$  gets large, then  $N$  asymptotically approaches zero. If however, there is immigration into the population, then the expected, time-averaged population density of a subpopulation will be given as:  $N^* = \frac{I}{|\langle f \rangle|}$ , which is always positive for  $I>0$ . A discrete time model of the same system (Roy et al. 2005), can be formulated as:  $N_{t+1} = R_t N_t + I$ . In constant conditions without immigration ( $I = 0$ ), the

asymptotic value of  $N$  is zero for  $0 < R < 1$ . But in the presence of variation, it is possible for a population to have a positive time-averaged population density:  $\overline{N} = \frac{Cov(R, N) + I}{1 - \overline{R}}$ , where

$Cov(R, N)$  is the temporal covariance between population size and growth rate, which, if  $0 < R < 1$ , and  $Cov(R, N)$  is positive, will produce a positive average asymptotic density. Of course, a negative correlation between  $R$  and  $N$  will yield a negative population density for  $0 < R < 1$ . Therefore, if  $R$  and  $N$  are positively correlated, then the expected population density of a sink population will be larger than that expected under constant conditions ('inflated'). The authors suggest that in positively autocorrelated environments, high population density is correlated with high per capita growth rates, and that this covariance boosts long term density.

For short finite periods, as noted by Puccia and Levins (1985), the covariance between population growth and density may be positive or negative. For example, if we plot a histogram of either covariance or mean population density (after 64 timesteps), we see that the average covariance between the per capita growth rate,  $\lambda$ , and population density is not necessarily elevated with an increase in positively autocorrelated variation (Fig. A4). Instead, there is an increase in both positive and negative average covariance with increased autocorrelation (i.e. populations are less likely to have average covariance close to zero). In addition, there is no necessary relationship between those populations with positive autocorrelation between growth rates and density, and those populations that successfully establish (i.e. populations with a positive autocorrelation between growth rate and density are more likely to establish, but not all populations do). Moreover, while it is true that for those populations with low  $\lambda$ , positive covariance will increase with the probability of establishment, it has no effect on persistence of these populations.

However, the findings of Holt and coauthors, and in particular the simulations explored in Roy et al. (2005), are not inconsistent with our results. These authors find increased average abundance of persistent sink metapopulation patches with increased variance and positive autocorrelation in growth rates, but note that these effects disappear without dispersal between patches. The 'outbreaks' to larger population size in these studies are related to autocorrelation in growth rates and positive covariance between growth rates and population size. As we show, the change in the distribution of covariance population growth and population size can lead to a larger number of populations with both large and small maximum population size in finite time (i.e. the distribution has heavy tails with large autocorrelation, Fig. A4).

In finite time, large autocorrelation produces a bimodal or extremely heavy tailed distribution of covariance, so that many populations also have a more negative correlation between growth rate and population density. When translated to the impact on population density, this bimodality is the same underlying explanation of the results that we report in the main text. While it

is not possible to have a population size less than zero, it is possible to greatly increase population size in a short interval. Therefore, the increased negative impacts of strong autocorrelation have no effect on population persistence, but the increased positive impacts greatly increase establishment probability.

## References

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