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## Appendix 1

### Model description (ODD of pi model v1.9.1)

#### *A general ontogenetic growth model for plants*

The essential basis for modeling the ontogeny of individual plants starts with an energy conservation equation (Enquist and Niklas 2001, West et al. 2001, Hou et al. 2008):

$$B_p = B_r + B_s = B_r + E_s dm / dt \quad (1)$$

where  $B_p$  is defined as total energy intake rate (i.e. gross photosynthetic rate). A fraction of this assimilated energy is consumed by respiration,  $B_r$ , the remainder is stored as reserves and used for synthesizing new tissues,  $B_s$  (Appendix 1 Fig. A1).  $E_s$  is the metabolic energy stored in one unit of biomass and  $dm/dt$  is the change in biomass ( $m$ ) per unit time ( $t$ ).

The rate of energy consumed by respiration,  $B_r$ , depends on three major processes that require energy (Appendix 1 Fig. A1): maintenance of biomass ( $B_{\text{maint}}$ ), ion transport ( $B_{\text{tran}}$ ) and biosynthesis ( $B_{\text{syn}}$ ), which can be summarized as (Lambers et al. 2008):

$$B_r = B_{\text{maint}} + B_{\text{tran}} + B_{\text{syn}} = \sum B_m m_{\text{living}} + B_{\text{tran}} + E_c dm / dt \quad (2)$$

where  $B_m$  denotes average mass-specific maintenance metabolic rate,  $m_{\text{living}}$  stands for the biomass of living tissues, and  $E_c$  designates the energy required to synthesize a unit of biomass.

Typical average biological parameters of plant cells (tissue) are taken as a fundamental unit here, and possible differences between tissues are ignored (West et al. 2001). Note that the terms  $B_s = E_s dm / dt$  in Eq. 1 and  $B_{\text{syn}} = E_c dm / dt$  in Eq. 2 are quite different:  $S$  stands for the rate of cumulative energy content of new biomass, whereas  $B_{\text{syn}}$  refers to the metabolic energy expended on biosynthesis which is dissipated as heat instead of obtained as stored biomass (Hou et al. 2008). Combining Eq. 1 and 2, we get

$$B_p = B_{\text{maint}} + B_{\text{tran}} + B_{\text{syn}} + B_s = \sum B_m m_{\text{living}} + B_{\text{tran}} + E_0 dm / dt \quad (3)$$

where  $E_0 = E_c + E_s$ , is constant for a given taxon and stands for the sum of energy stored in a unit of biomass plus the energy used to synthesize this biomass, i.e. the synthesis costs of a unit of biomass.

Equation 3 is quite general, but  $B_{\text{maint}}$  may vary between woody and non-woody plants, as woody plants contain nonliving tissues (e.g. heartwood in stem and root) which do not need energy for their maintenance (Enquist et al. 2009). Also large trees with a large amount of heartwood, contain much less living tissues ( $m_{\text{living}}$ ) in comparison to the total biomass ( $m$ ). We assume that during ontogeny, woody plants mainly expend energy for maintaining their photosynthetic tissues (leaves),  $m_L$ , and conducting tissues (standing sapwood of stem and root),  $m_C$ , and suppose

$$B_{\text{maint}} = \sum B_m m_{\text{living}} = B_m m_L + B_m m_C = B_L + B_C \quad (4)$$

for woody plants, where  $B_L$  and  $B_C$  specify the metabolic rate for maintaining photosynthetic and conducting tissues, respectively (Appendix 1 Fig. A1). Combining Eq. 3 and 4 we can get the energy conservation equation for woody plants:

$$B_p = B_m m_L + B_m m_C + B_{\text{tran}} + E_0 \quad dm / dt \quad (5)$$

Based on empirical measurements and theoretical assumptions linking biomass and metabolism, MST (West et al. 1999, 2009, Enquist 2002, Price et al. 2007, Enquist et al. 2009) predicts that whole-plant, or gross, photosynthesis rate,  $B_p$ , and ion transport metabolic rate  $B_{\text{tran}}$  allometrically scale with the total biomass of a plant,  $m$ , as  $B_p \propto B_{\text{tran}} \propto m^\theta$ , where  $\theta \equiv 1 / (2\alpha + \beta)$  and  $\alpha$  and  $\beta$  representing the geometry and biomechanics of the vascular network. Their values may vary across different taxa (Price et al. 2007).

Although the Eq. 5 can be easily recast by using empirical values of  $\alpha$  and  $\beta$ , we use  $\alpha = 1/2$  and  $\beta = 1/3$  as common and idealized cases here (Price et al. 2007, West et al. 2009), so that  $\theta = 3/4$ . Models based on these scaling relationships predict that the standing leaf biomass,  $m_L$ , scales with respect to total biomass as  $m_L \propto m^{3/4}$  across woody plants which was confirmed by empirical data (Sack et al. 2002, Niklas 2005).

The relationship between standing sapwood biomass ( $m_C$ ) and total biomass  $m$  is largely unknown, therefore here we assume that the tissue- or species-specific wood density of conducting tissues,  $d_C$ , is constant for a given plant. Its total volume of conducting tissues,  $v_C$ , can be formulated as  $v_C \propto A_S h$ , where  $A_S$  is the mean cross-sectional area of sapwood and  $h$  is the height of plant. Because  $A_S \propto m^{3/4}$  and  $h \propto m^{1/4}$  (Enquist 2002, Savage et al. 2010), we therefore derive the allometric relationship  $m_C = d_C v_C \propto A_S h \propto m^{3/4} m^{1/4} \propto m$  for woody plants. Substituting the allometric relationship on biomass for all related terms in Eq. 5 gives

$$B_0 m^{3/4} = B_m a_L m^{3/4} + B_m a_S m + a_{\text{tran}} m^{3/4} + E_0 \quad dm / dt \quad (6)$$

where  $B_p = B_0 m^{3/4}$  reflects the total energy intake rate (i.e. gross photosynthetic rate) under optimal situation,  $B_0$  is constant for a given taxon (West et al. 1999),  $a_L$ ,  $a_S$  and  $a_{\text{tran}}$  are normalization constants. Eq. 6 can therefore be rewritten as

$$dm/dt = a_1 m^{3/4} - b_1 m = a_1 m^{3/4} [1 - (m / M_1)^{1/4}] \quad (7)$$

with  $a_1 = (B_0 - B_m a_L - a_{\text{tran}}) / E_0$  and  $b_1 = B_m a_S / E_0$ . The value  $M_1 = (a_1 / b_1)^4$  is asymptotic maximum body size of the woody plant (calculated for  $dm / dt = 0$ ), which depends on species-specific traits and is determined by the systematic variation of the in vivo metabolic rate within different taxa (West et al. 2001). The gain term ( $a_1 m^{3/4}$ ) in Eq. 7 dominates while plants grow to a moderate size, which has been shown to be a good quantitative description of plant growth (Niklas and Enquist 2001, Enquist et al. 2009).

Across non-woody plants which lack secondary tissues (or juveniles of woody plant which have not accumulated much secondary tissue), the total biomass of living tissues (as leaves,  $m_L$ , stem,  $m_S$ , and roots,  $m_R$  is approximately equal to the whole plant mass,  $m_{\text{living}} = m_L + m_S + m_R \approx m$  (Enquist 2002). Combining Eq. 3 with those scaling relationship leads to

$$B_0 m^{3/4} = B_m m + a_{\text{tran}} m + E_0 dm / dt \quad (8)$$

for non-woody plants.

Taking the parameters in Eq. 8 in the same sense as before, Eq. 8 can be re-expressed as

$$dm / dt = a_2 m^{3/4} - b_2 m = a_2 m^{3/4} [1 - (m / M_2)^{1/4}] \quad (9)$$

with  $a_2 = B_0 / E_0$ ,  $b_2 = (B_m + a_{\text{tran}}) / E$ , and  $M_2 = (a_2 / b_2)^4$ , which is the asymptotic maximum body size of a non-woody plant.

Equation 7 and 9 have same form, and we use a general form of the growth function for both woody and non-woody plants:

$$dm / dt = am^{3/4} - bm = am^{3/4} [1 - (m / M_0)^{1/4}] \quad (10)$$

where  $a$  is a general constant and  $M_0$  is the generally asymptotic maximum body size of plant.

### *Plant growth under abiotic stress*

Because ‘stress’ is not a precise concept, the characteristics of abiotic stress factors are different and can be resource-independent or dependent (Maestre et al. 2009). We assume simply that abiotic stress factors act in two ways: restricting the energy intake rate or burdening the maintenance of plant or even concurrently, this is presumably always true for plants (Lambers et al. 2008). Since the plant growth rate is negatively and linearly related to the degree of abiotic stress (Travis et al. 2006, Chu et al. 2008, 2009), incorporating abiotic stress in Eq. 10, we have

$$dm / dt = am^{3/4} - bm - Sam^{3/4} = (1 - S)am^{3/4} [1 - (m / M_s)^{1/4}] \quad (11)$$

where  $S$  is a dimensionless efficiency factor that indicates the level of stress ranges from 0 (no stress) to 1 (extreme stress),  $Sam^{3/4}$  is the energy restricted or burdened by abiotic stress which is proportional to total energy intake rate and increasing with the degree of stress level, and  $M_s = (1 - S)^4 M_0$  is the maximum achievable biomass of plant under stress.

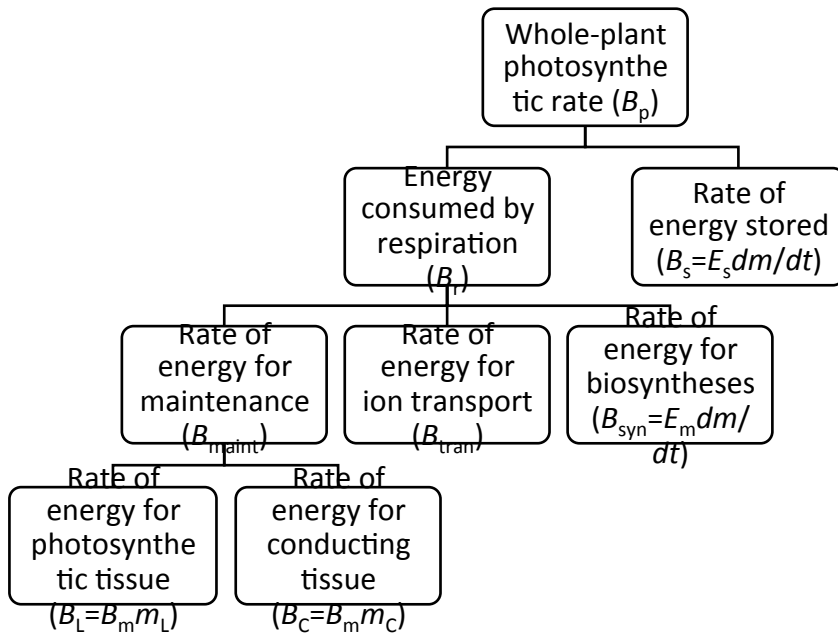


Figure A1. Assimilated energy partition of plants during ontogenetic growth.

## Individual-based model (IBM) considering mode of competition, facilitation, abiotic stress and ontogenetic growth

The following model description follows the ODD protocol (overview, design concepts, details) for describing individual- and agent-based models (Grimm et al. 2006, 2010),

### *Purpose*

The aim of this model is to evaluate the multiple effects of the modes of competition and facilitation under abiotic stress on regulating plant population dynamics, specifically on spatial pattern formation induced by density-dependent mortality. In particular, we test whether different modes of facilitations at individual plant level can result in the spatial aggregation of the population. The model does not represent specific species, but generic ones.

### *Entities, state variables, and scales*

The entities in the model are plants and square habitat units, or patches (Appendix 1 Table A1). Plants are characterized by the following state variables: initial growth rate, initial biomass, maximum biomass (asymptotic biomass), current biomass and their position, i.e. coordinates of the stem. Each individual plant has its own circular zone-of-influence (ZOI). The ZOI stands for the physical space occupied by a plant, and represents the energy and resources potentially available to

this plant, which is allometrically related to its body mass. Neighboring plants only compete for the resources when their ZOIs are overlapping.

In order to make the spatial calculations of resource competition easier, ZOIs are projected onto a grid of patches. To avoid edge effects, we use a torus world with a size of  $200 \times 200$  patches (Grimm and Railsback 2005). Each patch represents  $1 \text{ m}^2$  or  $1 \text{ cm}^2$  for woody- and non-woody plants, respectively. The state of each patch is characterized by its resource availability. We use a homogeneous environment here as all patches have the same, and constant, degree of abiotic stress. One time step in the model represents approximately one year for woody plants and one day for non-woody plants.

Table A1. State variables and initialization in the individual-based model. Actual values are drawn from the given intervals to introduce a certain degree of heterogeneity among individuals.

Variable	Description	Initial value [unit] (woody/non-woody)
Plants		
$c$	initial growth rate	$1 \pm 0.1 \text{ [kg m}^{-2} \text{ time step}^{-1}] / \text{[mg cm}^{-2} \text{ time step}^{-1}]$
$m_0$	initial body mass	$2 \pm 0.2 \text{ [kg]} / \text{[mg]}$
$M_0$	maximal biomass	$2 \times 10^6 \pm 2 \times 10^5 \text{ [kg]} / \text{[mg]}$
$m$	current biomass	$\text{[kg]} / \text{[mg]}$
$A$	zone of influence	$\text{[m}^{-2}] / \text{[cm}^{-2}]$
Patches		
$S$	abiotic stress level	0, 0.1, 0.5, 0.9
Initialization		
Mortality	threshold of death	5% of $m^{3/4}$
Density	no. of plants	256, 400, 676, 1089, 1849, 3025, 4900 $\text{[ha}^{-1}] / \text{[m}^{-2}]$
Random seed	generation of random number	1, 2, 3, 4, 5

### *Process overview and scheduling*

After initialization, all individual plants with a given density are randomly distributed in the world. The processes of local competition, facilitation, growth and mortality of each plant are fulfilled within each time step. In each step, individual plants first sense the environment qualities of patches within their ZOIs, the area (radius) of an individual plant's ZOI is determined by its current biomass. When their ZOIs are overlapping, individuals compete and facilitate within the overlapping area. Thus, the overlapping area reflecting resources is divided according to the mode of competition. At the same time, under abiotic stress in the presence of facilitation, the overlapping area reflecting the ameliorated habitat by neighbours is also divided according to the mode of

facilitation. Considering the outcome of the interaction process, all individual plants grow according to the growth function. Plants with growth rates falling below a threshold die and are removed immediately. The state variables of the plants are synchronously updated within the subroutines, i.e. changes to state variables are updated only after all individuals have been processed (Grimm and Railsback 2005).

### *Design concepts*

*Basic principles:* From ‘Metabolic scaling theory’, we derived a general ontogenetic growth model for individual plants. We combine this model, via the ZOI approach, with the effects of different modes of competition and facilitation under abiotic stress.

*Emergence:* All features observed at the population level, e.g. mass-density relationship or self-thinning trajectories (i.e. size distribution and spatial distribution, respectively), population size inequality and spatial pattern, are emerged from local interactions among plants under abiotic stress of their environment.

*Interaction:* Individual plants interact via competition for resources and facilitation from neighbours in the overlapping area of their ZOIs.

*Stochasticity:* Initial growth rate, initial biomass, maximum biomass and initial position of plants are randomly taken from the intervals given in Table 1. This introduces a certain level of heterogeneity among individual characteristics to take into account that real plants are never exactly identical.

*Observation:* Spatial point patterns of plants, population size, biomass of each plant, and mean biomass of all living plants are the main observations.

### *Initialization*

If the initial spatial pattern is aggregation, we randomly choose several patches (determined by the number of cohorts) as centers and to transplant groups of individual plants (initial density / number of cohorts) among the centers within a certain cluster diameter. In the case of randomness, individual plants are randomly distributed according to the chosen initial density. Resources and abiotic stress are spatially and temporally constant. Each plant has an initial biomass ( $m_0$ ), maximal biomass ( $M_0$ ) and initial growth rate ( $c$ ) drawn from truncated normal distributions with average and intervals given in Appendix 1 Table A1.

### *Input*

After initialization, the model does not include any external inputs, i.e. the abiotic environment is constant.

## Submodels

### Plant growth

In our individual-based model the plant's ZOI stands for the physical space occupied by a plant and represents the energy and resources potentially available to this plant. This space is allometrically related to the plant's body mass,  $m$ , as  $c_0 A = m^{3/4}$  (Enquist and Niklas 2001), where  $c_0$  is a normalization constant. Accordingly, Eq. 10 can be rewritten as

$$dm / dt = cA [1 - (m / M_0)^{1/4}] \quad (12)$$

and with abiotic stress, it becomes

$$dm / dt = (1 - S) cA [1 - (m / M_s)^{1/4}] \quad (13)$$

where  $c = ac_0$ , is the initial growth rates in units of mass per area and time interval. For simplicity, we choose  $c = 1 \pm 0.1$  in our model. In addition, we simulate the model with different  $c$  values. As expected, the results from different values were qualitatively similar (consist with our findings).

### Competition and facilitation under abiotic stress

Competition and facilitation are incorporated by using dimensionless factors or indexes,  $f_p$  and  $f_q$  respectively. With the above assumptions, Eq. 12 becomes:

$$dm / dt = f_p f_q cA [1 - (m / M)^{1/4}] \quad (14)$$

where  $M = (f_p f_q)^4 M_0$  is the maximum achievable biomass under stress with competition and facilitation.

As for competition, the modes of resource-mediated competition among plants can be located somewhere along a continuum between completely asymmetric competition (largest plants obtain all the contested resources) and completely symmetric competition (resource uptake is equal for all plants, independent of their relative sizes; Schwinning and Weiner 1998). To represent different modes of competition explicitly, we describe the competitive index  $f_p$  as

$$f_p = (A_{no} + \sum_{k=1}^{n_o} \frac{v_i m_i^p}{\sum_{j=1}^{n_j} v_j m_j^p} A_{o,k}) / A \quad (15)$$

This factor thus refers to the fraction of resources available in the area which plant  $i$  could obtain after a loss of potential resources due to areas overlapped by neighbours of sizes  $m_j$  (Schwinning and Weiner 1998).  $A_{no}$  is the area not overlapping with neighbors,  $A_{o,k}$  denotes the  $n_o$  areas overlapping with  $n_j$  different neighbors. Parameter  $p$  determines the mode of competition, ranging from complete symmetry ( $p = 0$ ) to complete asymmetry ( $p$  approaching infinity; for details and examples see Appendix 1 Fig. A2). In this research, we restrict ourselves to intraspecific facilitation and competition, and assume therefore the species-specific weighting constant of competition  $v_i$  and  $v_j$  equal 1 here, as conspecific case.

Simultaneously, assuming the effect of facilitation is additive (Molofsky 2001, Molofsky and Bever 2002, Chu et al. 2008, 2009), we define the effect of different modes of facilitation,  $f_q$ , as

$$f_q = 1 - \frac{S}{A_f + 1} = 1 - \frac{S}{\sum_{k=1}^{n_o} \left(1 - \frac{w_i m_i^q}{\sum_{j=1}^{n_j} w_j m_j^q}\right) A_{o,k} + 1} \quad (16)$$

This factor is based on SGH, which reflects the facilitative effect of relieving stress and is consist with other model (Chu et al. 2008, 2009, 2010, Jia et al. 2011; for details and examples see Appendix 1 Fig. A2). To keep things simple, we follow the assumption of Chu et al. (2008, 2009) that the ZOI,  $A$ , is the same for competition and facilitation (they could be different). Where  $A_f$  refers to the benefit gained by plant from all interactive neighbors, and is calculated as the sum of the areas (ZOIs) overlapped with neighbour plants. The index  $q$  determines the mode of facilitation among plants, ranging from complete symmetry ( $q = 0$ , algorithmic equivalent to the form used in Chu et al. 2008, 2009) to complete asymmetry ( $q$  approaching infinity; see Table 1 for the complete form and definitions). Since we investigate the monopopulation here, to simplify, we assume the species-specific weighting constant  $w$  equals 1 here as the conspecific case, so the facilitation is size dependent. When there is no facilitation ( $A_f = 0$ ), Eq. 16 becomes  $1 - S$ , which reflects the effect of abiotic stress.

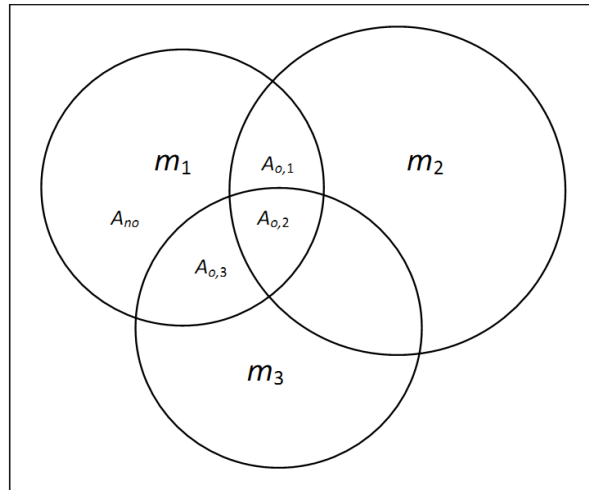


Figure A2. An example of calculating the interactive indexes (Eq. 15 and 16) with different modes of competition and facilitation by dividing plants' ZOI (zone-of-influence). Three plants with sizes  $m_1$ ,  $m_2$  and  $m_3$  are interacting in this example. For plant 1, its ZOI ( $A$ ) was divided into four parts:  $A_{no}$ , the area not overlapping with the other two plants;  $A_{o,1}$ , the area overlapping with plant 2;  $A_{o,2}$ , the area overlapping with plants 2 and 3;  $A_{o,3}$ , the area overlapping with plant 3.

Then the actual area that plant 1 can take from  $A_{o,1}$  is

$$A_{o,1} \frac{m_1^p}{\sum_{j=1}^2 m_j^p} = A_{o,1} \frac{m_1^p}{m_1^p + m_2^p}$$

For  $A_{o,2}$ ,

$$A_{o,2} \frac{m_1^p}{\sum_{j=1}^3 m_j^p} = A_{o,2} \frac{m_1^p}{m_1^p + m_2^p + m_3^p}$$

And for  $A_{o,3}$ ,

$$A_{o,3} \frac{m_1^p}{\sum_{j=1}^2 m_j^p} = A_{o,3} \frac{m_1^p}{m_1^p + m_3^p}$$

therefore, the competitive index for plant 1 is:

$$f_p = (A_{no} + A_{o,1} \frac{m_1^p}{m_1^p + m_2^p} + A_{o,2} \frac{m_1^p}{m_1^p + m_2^p + m_3^p} + A_{o,3} \frac{m_1^p}{m_1^p + m_3^p}) / A$$

Where  $A = m_1^{3/4} / c_0$ . For plant 1, the benefit received from neighbours is

$$A_f = A_{o,1} \frac{m_2^q}{m_1^q + m_2^q} + A_{o,2} \frac{m_2^q + m_3^q}{m_1^q + m_2^q + m_3^q} + A_{o,3} \frac{m_3^q}{m_1^q + m_3^q}$$

and the facilitative index for plant 1 under abiotic stress is:

$$f_q = 1 - \frac{S}{A_f + 1} = 1 - \frac{S}{A_{o,1} \frac{m_2^q}{m_1^q + m_2^q} + A_{o,2} \frac{m_2^q + m_3^q}{m_1^q + m_2^q + m_3^q} + A_{o,3} \frac{m_3^q}{m_1^q + m_3^q} + 1}$$

In total, Eq. 14 clearly shows how a plant's growth rate is jointly determined by abiotic stress,  $S$ , competition,  $f_p$ , and facilitation  $f_q$ . This also implies that a plant's final size is usually smaller than its asymptotic maximum size ( $M_0$ ) during environmental stress and local competition, but can increase by the beneficial effects of neighbour plants via the amelioration of habitat.

### Mortality

An individual's mortality rate is proportional to its mass-specific metabolism (as current total metabolic rate divide by body mass; Brown et al. 2004). Based on this, we assume that individuals die if their actual growth rate ( $dm / dt$ , realistic metabolic rate) falls below a threshold fraction of their basal metabolic rate (allometrically scaled with body mass), i.e. 5% of  $m^{3/4}$ . Therefore, individual plants may die due to metabolic inactivation driven by abiotic stress, competition, senescence (when  $m$  approaches  $M$ ) or combinations thereof. This provides a more realistic representation of relevant ecological process than in previous models (Stoll et al. 2002, Chu et al. 2009, 2010, Jia et al. 2011).

### Simulation experiment

The model was implemented in NetLogo. Simulation experiments were conducted by using "Behavior space", which is a tool of NetLogo for running simulations and collecting data. The

setting of our simulation experiments used in this study can be found in the .nlogo model file of Appendix 2. The simulation results of experiment “Simulation\_CV\_Exp-1” was showed in our paper.

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## Appendix 2

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## Appendix 3

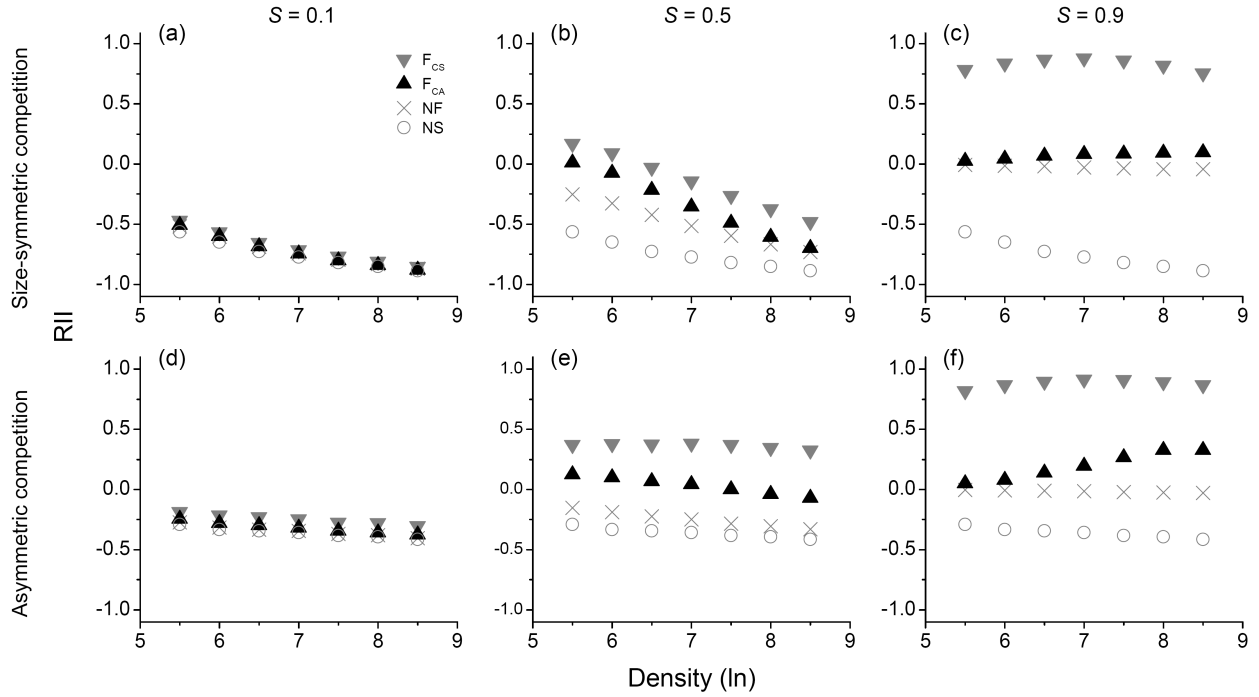


Figure A3. Relative interaction intensity (RII) of simulated plant populations without density-dependent mortality at different levels of abiotic stress (low stress,  $S = 0.1$ ; intermediate stress,  $S = 0.5$ ; high stress,  $S = 0.9$ ) after 30 time steps. Upper and lower panels show results for size-symmetric competition ( $p = 1$ ) and asymmetric competition ( $p = 10$ ), respectively.  $F_{CS}$ : completely symmetric facilitation ( $q = 0$ );  $F_{CA}$ : completely asymmetric facilitation ( $q = \infty$ ); NF: no facilitation ( $A_f = 0$ ); NS: benign condition without stress ( $S = 0$ ).