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Müller, D. W. H., Codron, D., Werner, J., Fritz, J., Hummel, J., Griebeler, E. M. and Clauss, M. 2012. Dichotomy of eutherian reproduction and metabolism. – Oikos 121: 102–115.

Appendix A1

Table A1. Sensitivity of linear (L) and quadratic (Q) scaling to sample size, from regressions fitted to 3×10^4 random subsamples derived from the mammal and bird basal MR datasets.

Total n	% of dataset extracted	n per subsample	Model	Mean R^2	Mean AIC	p (a)	p (b or b_1)	p (b_2)	p (best-fit)
Mammals									
637	10	64	L	0.9576	–112.310	<0.0001	<0.0001		0.4988
			Q	0.9601	–112.851	<0.0001	<0.0001	0.4218	0.5012
	25	159	L	0.9577	–285.089	<0.0001	<0.0001		0.6077
			Q	0.9606	–287.871	<0.0001	<0.0001	0.0650	0.3923
	50	319	L	0.9578	–569.115	<0.0001	<0.0001		0.9994
			Q	0.9608	–579.596	<0.0001	<0.0001	0.0002	0.0006
	75	478	L	0.9578	–854.634	<0.0001	<0.0001		
			Q	0.9609	–871.320	<0.0001	<0.0001	<0.0001	<0.0001
Birds									
530	10	53	L	0.9405	–109.481	<0.0001	<0.0001		0.0952
			Q	0.9403	–107.878	<0.0001	<0.0001	0.8585	0.9048
	25	133	L	0.9411	–277.165	<0.0001	<0.0001		0.1718
			Q	0.9414	–275.973	<0.0001	<0.0001	0.7579	0.8282
	50	265	L	0.9413	–556.608	<0.0001	<0.0001		0.2674
			Q	0.9417	–555.990	<0.0001	<0.0001	0.6258	0.7326
	75	398	L	0.9414	–836.061	<0.0001	<0.0001		0.3749
			Q	0.9418	–835.938	<0.0001	<0.0001	0.4183	0.6251

This table shows that a linear allometric fit and its parameters remain significant irrespective of the size and M range of the sample, but the parameter b_2 of a quadratic fit becomes less significant at smaller sample sizes. Accordingly, in smaller datasets, the relative strength of quadratic over linear models is likely to be lost, even in cases where b_2 retains its significance.

Birds are an extreme example: in the whole dataset, a quadratic function provides a slightly better fit compared with a linear function, but this preference rapidly subsides, as does the significance of the polynomial term, in smaller data subsets.

These results suggest that sample size may influence statistical power, particularly of quadratic regressions, when comparing linear with non-linear scaling in allometry. We demonstrate below that these effects of sample size are, however, not entirely an effect of reduced statistical power, but are more likely influenced by the range of M included in a dataset.

The figures below show the influence of body mass (M) distributions on linear and quadratic fits. M distributions were manipulated in the mammal (Part 1) and bird (Part 2) basal MR datasets by random resampling. We explore the effect of M range and mean M, and of the minimum and maximum M point included in a dataset, on three statistics related to a comparison of linear (L) with quadratic (Q) scaling: significance of the allometric exponent (*b*), significance of the polynomial term (b_2), and the evidence to support a better fit of Q to L (lower AIC score for Q). The mean ± 1 SE for each of these statistics, derived from 3×10^4 permutations, are shown.

In both datasets, the parameters *a*, *b* and b_1 for the respective equations were consistently significant, but the polynomial term b_2 was not. In mammals, a linear increase of b_1 with M on a logarithmic scale is evident (Fig. A1.2a), and is a condition supporting that any curvature in allometry solves to a quadratic polynomial (Eq. 3–5 in the main text). However, significance of the polynomial term is only evident when the M range is at least 4, possibly 5, orders of magnitude (Fig. A1.1b), and this condition may be missing in smaller datasets. Similarly, whereas the polynomial term and quadratic fit is unanimously favored for mammal BMR in larger datasets, this support is reduced in smaller datasets, for example if the data do not include species below $\log(M) \approx -2.0$ (i.e. 0.01 kg, or 10 g; Fig. A1.3a), or excludes species above $\log(M) \approx 3.0$ (i.e. 1000 kg; Fig. A1.4a). In summary, larger datasets are likely to include a wide range of M, from where quadratic scaling would be evident, but in datasets excluding species < 0.01 kg and/or > 1000 kg, quadratic scaling is unlikely to be detected. This indicates that quadratic scaling is an artefact of changes in allometry at the extreme ends of the M range. For birds, similar rules for the detection of quadratic scaling could be found: simply, scaling appears to be linear except in the entire dataset, strongly indicating a spurious result for the significance of a quadratic fit.

Supplementary figures Part 1: Mammal BMR

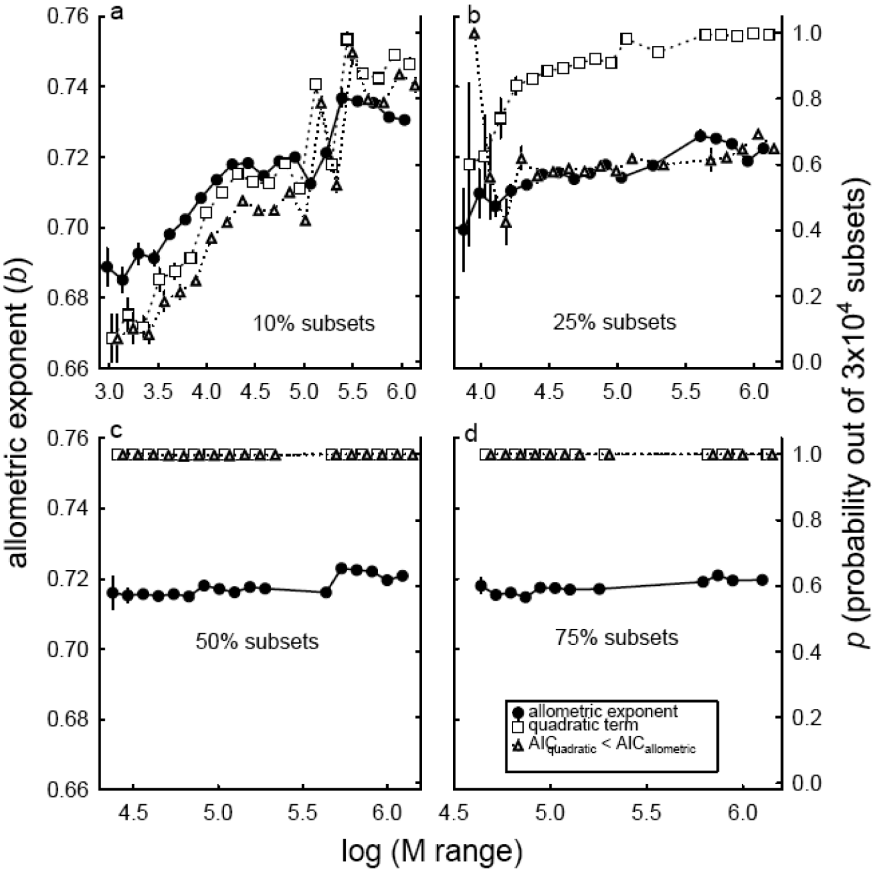

Figure A1.1. Response of 1) the simple allometric exponent *b* (solid circles); 2) probability of a significant ($p < 0.05$) quadratic term (open squares); and 3) probability that the quadratic function provides a better-fit to log-transformed data than the linear function (probability of lower AIC score in the former) (open triangles), to increases in the range of body mass (M, kg) included in the data (maximum – minimum M). M ranges were manipulated by random subsamples representing 10% (a), 25% (b), 50% (c) or 75% (d) of the data, with 3×10^4 permutations.

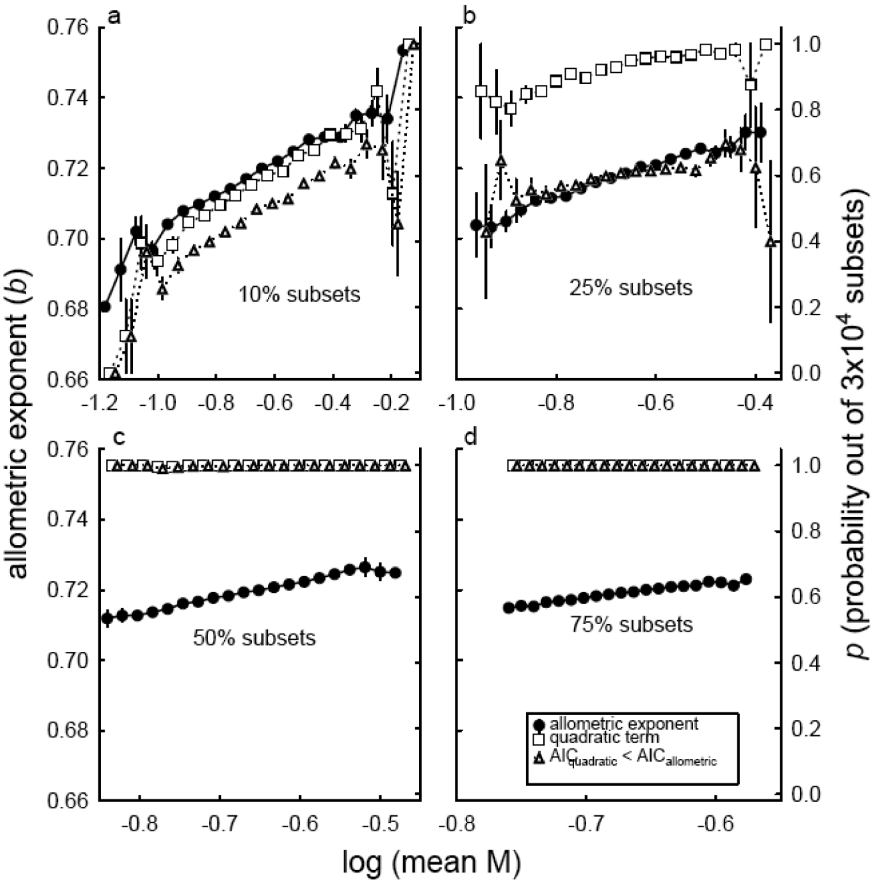

Figure A1.2. Response of 1) the simple allometric exponent *b* (solid circles); 2) probability of a significant ($p < 0.05$) quadratic term (open squares); and 3) probability that the quadratic function provides a better-fit to log-transformed data than the linear function (probability of lower AIC score in the former) (open triangles), to increases in the mean body mass (M, kg) included in the data. M ranges were manipulated by random subsamples representing 10% (a), 25% (b), 50% (c) or 75% (d) of the data, with 3×10^4 permutations.

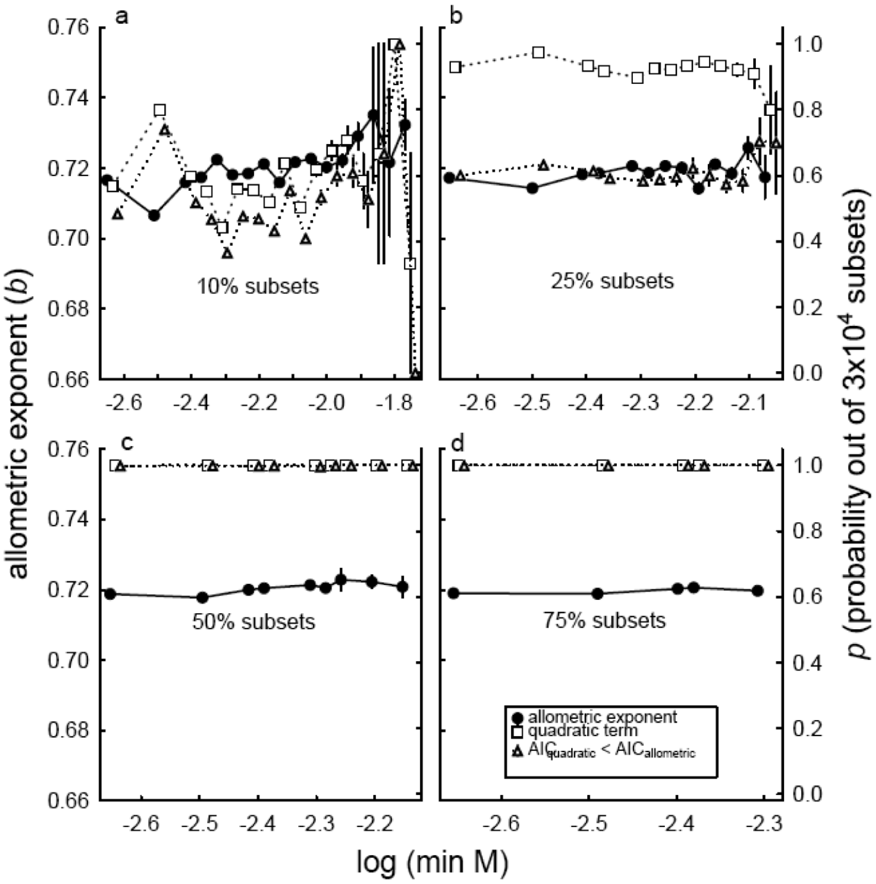

Figure A1.3. Response of 1) the simple allometric exponent *b* (solid circles); 2) probability of a significant ($p < 0.05$) quadratic term (open squares); and 3) probability that the quadratic function provides a better-fit to log-transformed data than the linear function (probability of lower AIC score in the former) (open triangles), to increases in the minimum body mass (M, kg) included in the data. M ranges were manipulated by random subsamples representing 10% (a), 25% (b), 50% (c) or 75% (d) of the data, with 3×10^4 permutations.

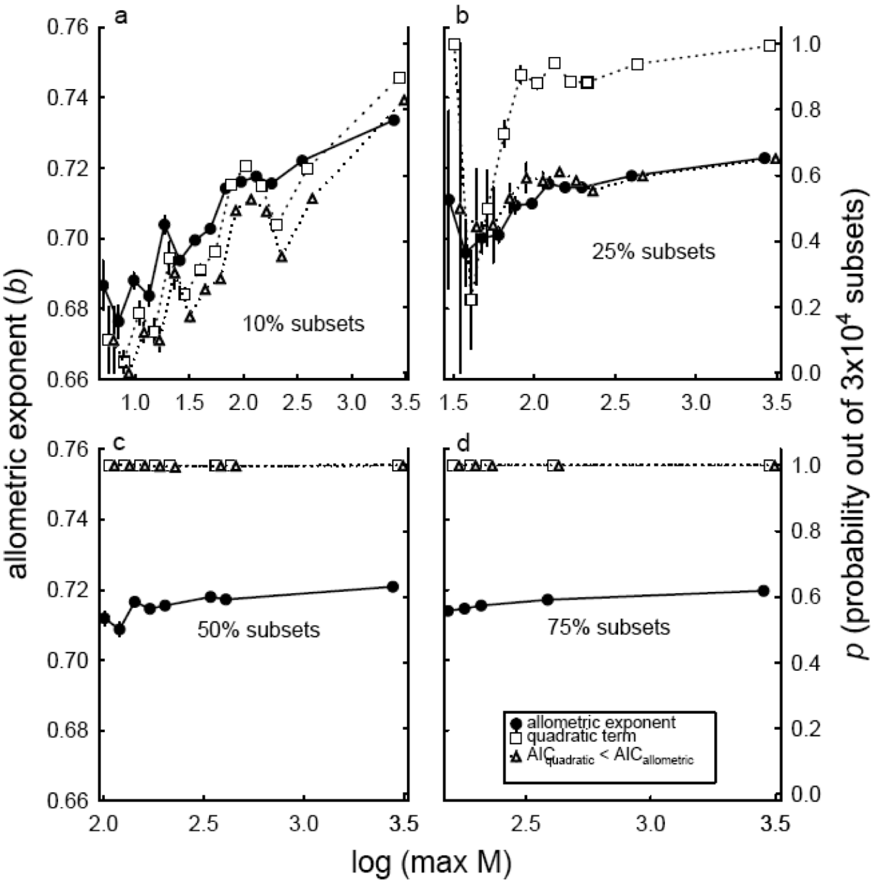

Figure A1.4. Response of 1) the simple allometric exponent *b* (solid circles); 2) probability of a significant ($p < 0.05$) quadratic term (open squares); and 3) probability that the quadratic function provides a better-fit to log-transformed data than the linear function (probability of lower AIC score in the former) (open triangles), to increases in the maximum body mass (M, kg) included in the data. M ranges were manipulated by random subsamples representing 10% (a), 25% (b), 50% (c) or 75% (d) of the data, with 3 × 10^4 permutations.

Supplementary figures Part 2: Bird BMR

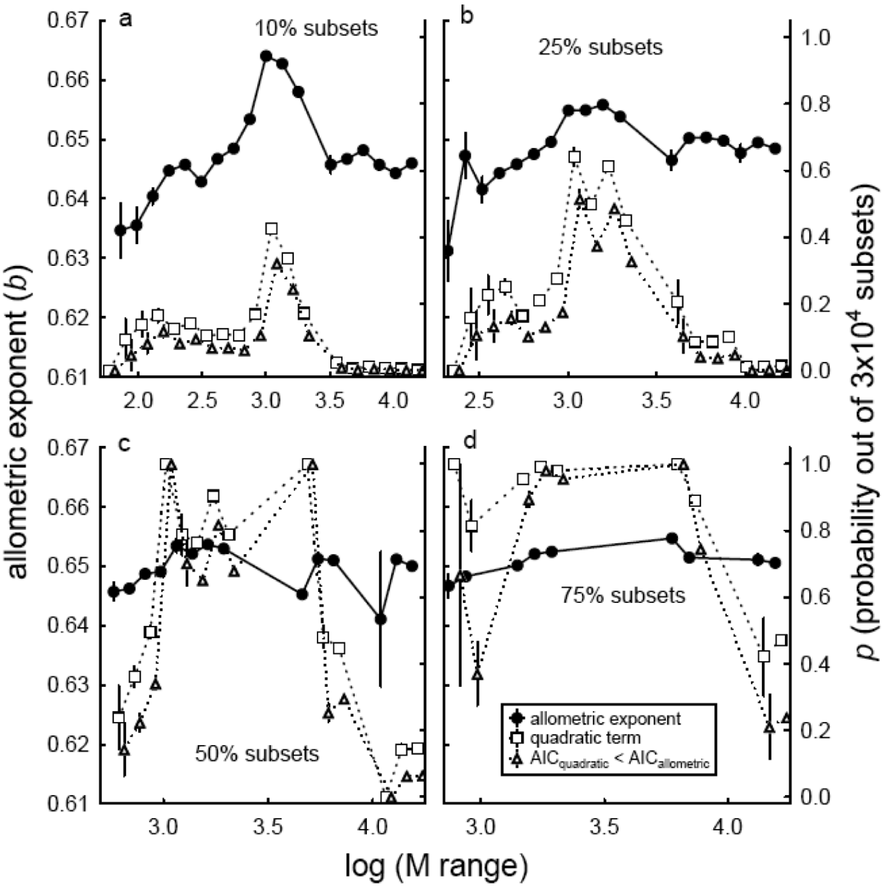

Figure A2.1. Response of 1) the simple allometric exponent *b* (solid circles); 2) probability of a significant ($p < 0.05$) quadratic term (open squares); and 3) probability that the quadratic function provides a better-fit to log-transformed data than the linear function (probability of lower AIC score in the former) (open triangles), to increases in the range of body mass (M, kg) included in the data (maximum – minimum M). M ranges were manipulated by random subsamples representing 10% (a), 25% (b), 50% (c) or 75% (d) of the data, with 3×10^4 permutations.

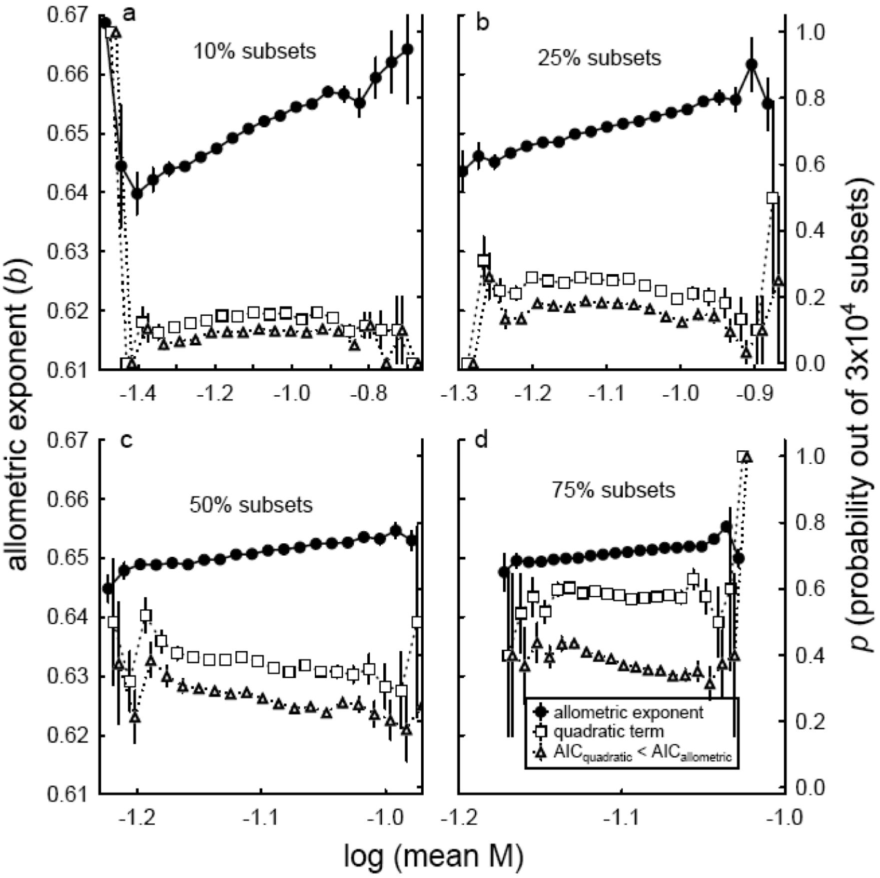

Figure A2.2. Response of 1) the simple allometric exponent *b* (solid circles); 2) probability of a significant ($p < 0.05$) quadratic term (open squares); and 3) probability that the quadratic function provides a better-fit to log-transformed data than the linear function (probability of lower AIC score in the former) (open triangles), to increases in the mean body mass (M, kg) included in the data. M ranges were manipulated by random subsamples representing 10% (a), 25% (b), 50% (c) or 75% (d) of the data, with 3×10^4 permutations.

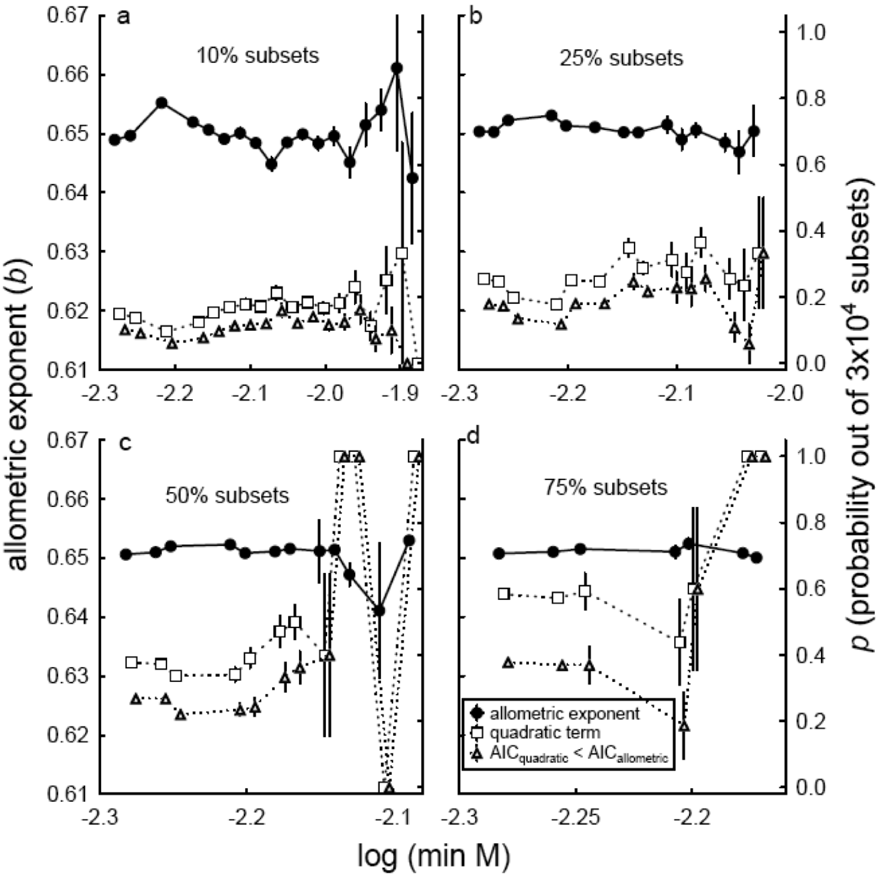

Figure A2.3. Response of 1) the simple allometric exponent *b* (solid circles); 2) probability of a significant ($p < 0.05$) quadratic term (open squares); and 3) probability that the quadratic function provides a better-fit to log-transformed data than the linear function (probability of lower AIC score in the former) (open triangles), to increases in the minimum body mass (M, kg) included in the data. M ranges were manipulated by random subsamples representing 10% (a), 25% (b), 50% (c) or 75% (d) of the data, with 3×10^4 permutations.

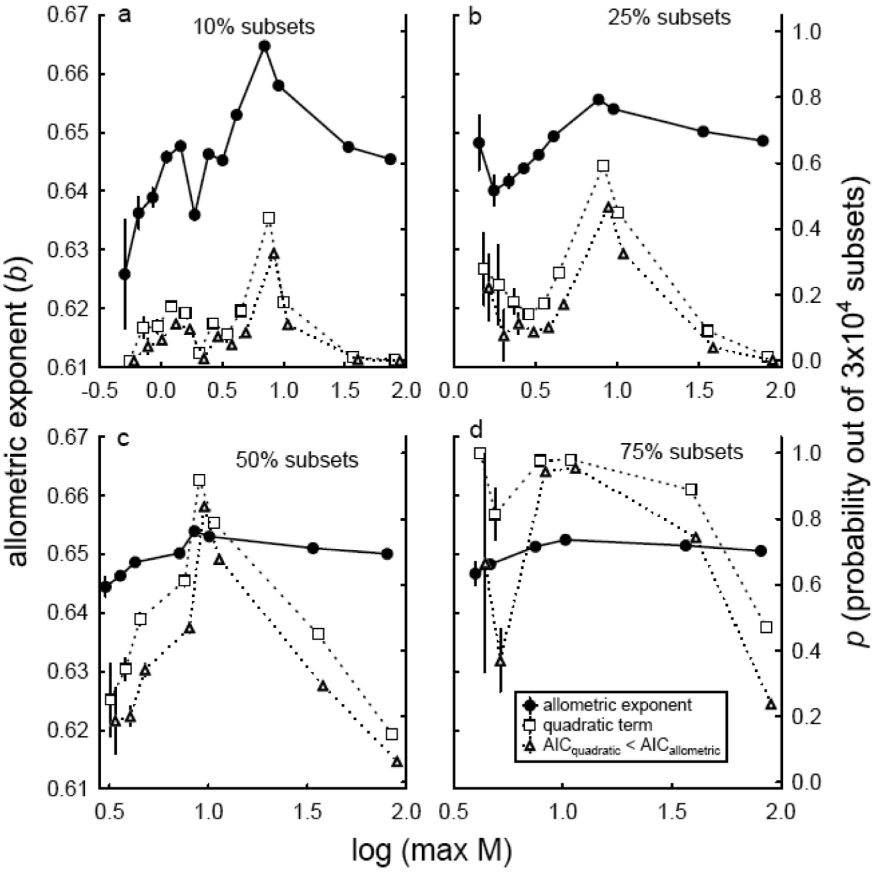

Figure A2.4. Response of 1) the simple allometric exponent *b* (solid circles); 2) probability of a significant ($p < 0.05$) quadratic term (open squares); and 3) probability that the quadratic function provides a better-fit to log-transformed data than the linear function (probability of lower AIC score in the former) (open triangles), to increases in the maximum body mass (M, kg) included in the data. M ranges were manipulated by random subsamples representing 10% (a), 25% (b), 50% (c) or 75% (d) of the data, with 3×10^4 permutations.

Table A2a. Comparison of linear (L) and quadratic (Q) regressions for raw data of anatomical and physiological traits on body mass (kg) in mammals. All variables were log-transformed prior to analysis. Note that the quadratic term was not significant in any case.

Trait	Shape	n	*a*	–95% CI	+95% CI	*b* or b_1	–95% CI	+95% CI	b_2	–95% CI	+95% CI
Organ mass											
Heart	L	82	–2.2123	–2.2575	–2.1671	0.9754	0.9454	1.0055			
	Q	82	–2.2175	–2.2648	–2.1703	0.9624	0.9171	1.0076	0.0076	–0.0121	0.0274
Kidney	L	74	–2.1452	–2.1918	–2.0987	0.8686	0.8381	0.8990			
	Q	74	–2.1473	–2.1969	–2.0978	0.8641	0.8182	0.9099	0.0027	–0.0175	0.0228
Liver	L	76	–1.4902	–1.5306	–1.4498	0.8998	0.8733	0.9263			
	Q	76	–1.4889	–1.5317	–1.4461	0.9026	0.8628	0.9424	–0.0017	–0.0192	0.0159
Lung	L	76	–1.9663	–2.0292	–1.9034	1.0141	0.9724	1.0559			
	Q	76	–1.9843	–2.0496	–1.9189	0.9730	0.9113	1.0347	0.0242	–0.0028	0.0511
GIT	L	32	–1.0855	–1.2114	–0.9596	1.0150	0.9457	1.0843			
	Q	32	–1.0863	–1.2201	–0.9524	1.0133	0.8986	1.1280	0.0009	–0.0439	0.0456
Respiratory and circulation											
Lung volume	L	32	1.6668	1.6195	1.7141	1.0549	1.0273	1.0825			
	Q	32	1.7025	1.6299	1.7750	1.0587	1.0307	1.0866	–0.0125	–0.0320	0.0069
Lung alvolar surface	L	32	0.5363	0.4844	0.5882	0.9358	0.9055	0.9661			
	Q	32	0.4940	0.4148	0.5732	0.9313	0.9008	0.9618	0.0149	–0.0064	0.0362
Breathing frequency	L	53	1.7410	1.6614	1.8206	–0.2379	–0.2793	–0.1965			
	Q	53	1.7326	1.6489	1.8164	–0.2560	–0.3237	–0.1883	0.0091	–0.0178	0.0360
Heart rate	L	23	2.3466	2.2945	2.3987	–0.2034	–0.2305	–0.1764			
	Q	23	2.3513	2.2955	2.4072	–0.1944	–0.2376	–0.1511	–0.0041	–0.0190	0.0109

Parameters *a*, b_1, b_2 correspond with Eq. 2 and 5, respectively, of the main text

Table A2b. Comparison of linear (L) and quadratic (Q) regressions for phylogenetic generalized least-squares of anatomical and physiological traits on body mass (kg) in mammals. All variables were log-transformed prior to analysis. Note that the quadratic term was not significant in any case.

Trait	Shape	*a*	–95% CI	+95% CI	*b* or b_1	–95% CI	+95% CI	b_2	–95% CI	+95% CI
Organ mass										
Heart	L	–2.2454	–2.3881	–2.1027	0.9465	0.9120	0.9810			
	Q	–2.2488	–2.3921	–2.1055	0.9377	0.8854	0.9900	0.0047	–0.0159	0.0253
Kidney	L	–2.1508	–2.2941	–2.0075	0.8749	0.8414	0.9084			
	Q	–2.1594	–2.3021	–2.0167	0.8561	0.8055	0.9067	0.0098	–0.0102	0.0298
Liver	L	–1.4631	–1.5558	–1.3704	0.8941	0.8659	0.9223			
	Q	–1.4734	–1.5671	–1.3797	0.8696	0.8269	0.9123	0.0132	–0.0040	0.0304
Lung	L	–1.9589	–2.0728	–1.8450	1.0024	0.9544	1.0504			
	Q	–1.9625	–2.0142	–1.9108	0.9764	0.9058	1.0470	0.0215	–0.0083	0.0513
GIT	L	–0.9959	–1.1400	–0.8518	0.9803	0.9070	1.0536			
	Q	–0.9937	–1.3396	–0.6478	1.2211	1.1719	1.2703	–0.0454	–0.0932	0.0024
Respiratory and circulation										
Lung volume	L	1.6513	1.5468	1.7558	1.0114	0.9687	1.0541			
	Q	1.6691	1.5568	1.7814	1.0188	0.9729	1.0647	–0.0080	–0.0270	0.0110
Lung alvolar surface	L	0.5086	0.3761	0.6411	0.9174	0.8666	0.9682			
	Q	0.4732	0.3291	0.6173	0.9032	0.8495	0.9569	0.0159	–0.0049	0.0367
Breathing frequency	L	1.7532	1.6658	1.8406	–0.2414	–0.2845	–0.1983			
	Q	1.7406	1.6783	1.8029	–0.2557	–0.3116	–0.1998	0.0099	–0.0146	0.0344
Heart rate	L	2.3541	2.2831	2.4251	–0.2087	–0.2381	–0.1793			
	Q	2.3671	2.3485	2.3857	–0.1834	–0.2057	–0.1611	–0.0083	–0.0187	0.0021

Parameters *a*, b_1, b_2 correspond with Eq. 2 and 5, respectively, of the main text