Verdú, M. and Valiente-Banuet, A. 2011. The relative contribution of abundance and phylogeny to the structure of plant facilitation networks. – Oikos 120: 1351-1356
Appendix A. Procedure to transform the probability matrices for improving the likelihood of the model explaining the observed facilitation interactions.

ABUNDANCE

The underlying assumption of the model is that the probability of interaction between two plants increases with their abundances in the community. This assumption is correct, as shown by Fig S1 in which pairwise values of the observed interaction matrix were plotted against the values of the abundance matrix.

![Graph showing relationship between pairwise plant abundance products and facilitation interactions.](image)

Fig S1. The observed facilitation interactions increase with plant abundances.

We considered three different theoretical scenarios in which the probability of interaction between two plants increases with plant abundances following a i) linear, ii) power curve or iii) exponential.

For the linear relationship \((facilitation \sim a \times abundance)\), we calculated the AIC of the likelihood function of Vázquez et al. (2009) without transforming the abundance matrix.

\[AIC(\text{linear}) = 7158\]
For the power relationship (\(facilitation \sim a \times abundance^b\)), we calculated the AIC of the likelihood function of Vázquez et al. (2009) after raising the abundance matrix to a given exponent \(b\). We searched the optimum \(b\) – that minimizing the AIC of the likelihood function - in the parameter space between 0 and 1 by small increments of \(1e-05\) (Fig S2).

![AIC of the power curve models](image)

**FIGS2.** AIC of the power curve models (\(facilitation \sim a \times abundance^b\)) for different \(b\) values.

The minimum AIC \((AIC=6266)\) corresponds to a model in which \(b=0.6786\).

\[AIC\text{(power)}=6264\]

Although this model requires estimating one parameter more than the linear model, it has a better AIC (6264) and improves very much the fit of the model.

For the exponential relationship (\(facilitation \sim b \times \exp(m \times abundance)\)), we calculated the AIC of the likelihood function of Vázquez et al. (2009) after exponentiation of the abundance matrix multiplied by a given \(m\). We searched the optimum \(m\) – that minimizing the AIC of the likelihood function - in the parameter space between 0 and 0.1 by small increments of \(1e-06\) (Fig S3).
FIGS3. AIC of the exponential models ($facilitation \sim b \times \exp(m \times abundance)$) for different $m$ values.

The minimum AIC ($AIC=8937$) corresponds to a model in which $m=0.00108$

$AIC(\text{exponential})=8937$

This model performed worse than the power and linear models.

------------------------------------------------------------------------

PHYLOGENY

For the matrix of phylogenetic distances, we also checked that the probability of interaction between two plants increases with their phylogenetic distances (Fig S4).
Fig S4. The observed facilitation interactions increase with the phylogenetic distance between plants.

We considered three different theoretical scenarios in which the probability of interaction between two plants increases with plant phylogenetic distances following a i) linear, ii) power curve or iii) exponential.

For the linear relationship (facilitation $\sim a \times$ phylogeny), we calculated the AIC of the likelihood function of Vázquez et al. (2009) without transforming the phylogeny matrix.

$AIC(\text{linear})=11195$

For the power relationship (facilitation $\sim a \times$ phylogeny$^b$), we calculated the AIC of the likelihood function of Vázquez et al. (2009) after raising the phylogeny matrix to a given exponent $b$. We searched the optimum $b$ – that minimizing the AIC of the
likelihood function- in the parameter space between 0 and 1 by small increments of 1e-05 (Fig S5).

FIGS5. AIC of the power curve models \( (facilitation \sim a \times phylogeny^b) \) for different \( b \) values.

The minimum AIC \( (AIC=10934) \) corresponds to a model in which \( b=0.28781 \).

\[
AIC(\text{power}) = 10934
\]

Although this model requires estimating one parameter more than the linear model, it has a better AIC and improves very much the fit of the model.

For the exponential relationship \( (facilitation \sim b \times \exp(m \times phylogeny)) \), we calculated the AIC of the likelihood function of Vázquez et al. (2009) after exponentiation of the phylogeny matrix multiplied by a given \( m \). We searched the optimum \( m \) –that minimizing the AIC of the likelihood function- in the parameter space between 0 and 0.1 by small increments of 1e-06 (Fig S6).
FIGS6. AIC of the exponential models ($facilitation \sim b \times \exp(m \times phylogeny)$) for different $m$ values.

The minimum AIC ($AIC=10843$) corresponds to a model in which $m=0.00248$

$AIC_{(exponential)}=10843$.

This model outperforms the linear and power models.

ABUNDANCE X PHYLOGENY

The model using the product of the transformed abundance ($abundance^{0.6786}$) and phylogeny ($\exp(0.00248*phylogeny)$) matrices improves very much the AIC of the model using the product of untransformed matrices (6020 vs. 7162).
Appendix 2

Download Appendix2.xls