Appendix 1

(A) Theoretical derivation of the GGE and TERCP as a function of body size

(1) NGE for energy as a function of body size
Rem: the symbols used are the same as in Hou et al. (2008) and are thus only briefly defined here. Interested readers are referred to this article if they want to learn more about the meaning of the symbols and the assumptions of the model.

A first step in the process is to express NGE (net growth efficiency for energy) as a function of body mass:

\[ \text{NGE}_{E} = \frac{S}{A} \]  

where \( S \) is the rate of storage of energy in the biomass by a growing organism and \( A \) is the rate of energy assimilation from food.

\( S \) can be expressed as:

\[ S = \gamma B_{\text{syn}} \]  

where \( B_{\text{syn}} \) is the rate of energy used to synthesize new biomass, but not stored into biomass (energetic cost of growth).

\( \gamma \) is the ratio of the energy stored in a unit of biomass to the energy expended to synthesize this biomass.

In turn, \( B_{\text{syn}} \) can be expressed as:

\[ B_{\text{syn}} = B_{\text{rest}} - B_{\text{maint}} \]  

where \( B_{\text{rest}} \) is the maintenance metabolic rate and \( B_{\text{rest}} \) is the resting metabolic rate (includes the energy rates for maintenance, biomass synthesis and digestion).

Both these rates are related to body size and temperature (outside temperature for poikilotherms and internal temperature for homeotherms):

\[ B_{\text{rest}} = B_{0} m^{3/4} e^{E_r k T} \]  

where \( B_0 \) is constant for a given taxon, \( m \) is the biomass as an instant \( t \), \( E_r \) is the energy activation constant (~0.65 eV), \( k \) is Boltzmann constant (8.62 \( \times \) 10^{-5} eV/K) and \( T \) is temperature in Kelvin.

\[ B_{\text{maint}} = B M^{1/4} e^{E_r k T} \]  

where \( B M^{1/4} e^{E_r k T} \) is the mass-specific maintenance metabolic rate and \( M \) is the asymptotic or adult body mass for organisms with determinate growth or indeterminate growth respectively.

Equation 2, 4 and 5 combined yield an expression for the relation between \( S \) and body mass:

\[ S = \gamma B_0 m^{3/4} \left( \frac{m}{M} \right)^{1/4} e^{E_r k T} \]  

On the other hand, \( A \) can be rewritten as:

\[ A = B_{\text{rest}} + B_{\text{act}} + S \]  

where \( B_{\text{rest}} \) is the rate of energy spent on locomotion, feeding and other activities.

\( B_{\text{act}} \) is assumed to be a multiple of \( B_{\text{rest}} \) with a factor:

\[ B_{\text{act}} = (f - 1)B_{\text{rest}} \]  

where \( f \) is the activity scope.

Equation 2, 3, 7 and 8 combined yield:

\[ A = (f - 1)B_{\text{rest}} + \gamma B_{\text{maint}} \]  

The final step to yield the body mass dependence of \( A \) is to replace \( B_{\text{syn}} \) and \( B_{\text{maint}} \) by their expressions in Eq. 4 and 5:

\[ A = B_{0} M^{1/4} \left( (f + \gamma) \frac{m}{M} - \gamma \frac{m}{M} \right) e^{E_r k T} \]  

NGE is the ratio of \( S \) to \( A \):

\[ \text{NGE}_{E} = \frac{S}{A} = \frac{\gamma B_0 m^{3/4} \left( \frac{m}{M} \right)^{1/4} e^{E_r k T}}{B_{0} M^{1/4} \left( (f + \gamma) \frac{m}{M} - \gamma \frac{m}{M} \right) e^{E_r k T}} \]  

Pulling some factors out of the numerator and denominator and a few simplifications yield the final expression for \( \text{NGE}_{E} \):

\[ \text{NGE}_{E} = 1 - \frac{f}{f + \gamma - \gamma \left( \frac{m}{M} \right)^{1/4}} \]  

A few comments on Eq. 12:
Apart from instantaneous and adult body masses, \( \text{NGE}_{E} \) depends on only two other physiological parameters that are not consistently related to body size nor temperature, the activity scope \( f \) and the ratio of the energy stored to the energy expended in biomass \( \gamma \).

\( \text{NGE}_{E} \) does not depend on temperature. It does depend on the ratio of instantaneous body mass to adult body mass: it reaches its
maximum value \(1 - \frac{f}{f + \gamma}\) when \(m/M\) is almost 0 (i.e. \(m\) is negligible in comparison to \(M\)); it decreases with an increase in \(m/M\); and is equal to 0 when \(m\) is equal to \(M\) (i.e. when the organism reaches its adult body mass), thus reflecting the stop in growth that occurs at the adult stage.

(2) GGE for energy as a function of body size

GGE is the NGE multiplied by assimilation efficiency (AE). Hence:

\[
GGE = \text{AE} \times \text{NGE}
\]

There is no clear expectation in the literature on the relationship between \(\text{AE}\) and body size during ontogenetic growth. Experimental data show mixed patterns, with some experiments documenting a significant effect of body size on \(\text{AE}\) (Urabe and Watanabe 1991) and others no effect (Peck et al. 2003). In absence of any clear prediction, we assume that there should not be any consistent relationship between \(\text{AE}\) and body size. As a result, \(GGE\) should show the same size-dependence as \(\text{NGE}\):

\[
GGE = \text{AE} \times \left(1 - \frac{f}{f + \gamma - \gamma \left(\frac{m}{M}\right)^{1/4}}\right)\tag{14}
\]

(3) GGE for carbon as a function of body size

\(\text{GGE}\) is the ratio of energy stored in biomass (\(S_i\)) to energy ingested (\(I_i\)):

\[
GGE = \frac{S_i}{I_i}\tag{15}
\]

To retrieve \(\text{GGE}\), one can multiply \(S_i\) by \(C_i^{\text{E}},\) the ratio of carbon-to-energy in biomass, and \(I_i\) by \(C_i^{\text{E}}\), the carbon-to-energy ratio in the food ingested:

\[
\text{GGE} = \text{GGE} \times \frac{C_i^{\text{E}}}{E_i} / \frac{C_i^{\text{E}}}{E_i}\tag{16}
\]

There should be no consistent relationship between carbon-to-energy ratios and body size, resulting in a size-dependence of \(\text{GGE}\) similar to \(\text{GGE}\). For higher trophic levels, \(C_i^{\text{E}},\) and \(C_i^{\text{E}}\) should be close, so that \(\text{GGE}\) and \(\text{GGE}\) should be almost equal.

(4) GGE for nitrogen as a function of body size

\(\text{GGE}\) can be calculated from \(\text{GGE}\) in a similar way to \(\text{GGE}\), by multiplying it by \(N_i^{\text{E}},\) the ratio of nitrogen-to-energy in biomass, and dividing it by \(N_i^{\text{E}}\), the nitrogen-to-energy ratio in the food ingested:

\[
\text{GGE} = \text{GGE} \times \frac{N_i^{\text{E}}}{E_i} / \frac{N_i^{\text{E}}}{E_i}\tag{17}
\]

As for carbon, we assume that there is no significant effect of body size on \(N_i^{\text{E}}\) and \(N_i^{\text{E}}\). Hence, \(\text{GGE}\) should follow the same size-dependency as \(\text{GGE}\).

(5) GGE for phosphorus as a function of body size

\(\text{GGE}\) can be calculated from \(\text{GGE}\):

\[
\text{GGE} = \text{GGE} \times \frac{P_i}{E_i} / \frac{P_i}{E_i}\tag{18}
\]

where \(P_i^{\text{E}}\) is the ratio of phosphorus-to-energy in biomass, and \(P_i^{\text{E}}\), the phosphorus-to-energy ratio in the food ingested. \(P_i^{\text{E}}\) should not vary with body size in experiments if food is set by experimenters.

(6) TERC as a function of body size

TERC\(_{\text{C.P}}\) can be written as:

\[
\text{TERC}_{\text{C.P}} = \left(\frac{\text{GGE}}{\text{GGE}}\right) \times \frac{C}{P} = \left(\frac{\text{GGE}}{\text{GGE}}\right) \times \frac{C/m}{P/m}\tag{22}
\]

Since \(\text{GGE}\) is proportional to \(P/m \times \text{GGE}\) (Eq. 18, 19 and 21), the size-dependence of these three factors should cancel out. Since \(C/m\) is not consistently related to body size, then \(\text{TERC}_{\text{C.P}}\) should not be affected by body size.
(B) Empirical relationship between relative mass (ratio of measured weight to asymptotic weight) and asymptotic weight in our dataset

The previous section shows that the various elemental GGE vary as a function of m/M, the ratio of measured weight to asymptotic weight. In our study, we are interested in the relationship between asymptotic body size, M, and the various elemental GGE. If there is a correlation between m/M and M in our dataset, whatever the underlying cause is, then we can predict a significant correlation between M and the various elemental GGEs, simply based on this correlation between M and the causal variable m/M.

In our dataset, the relationship between m/M and M is significant for invertebrates and for multicellular organisms (Fig. 1). The correlation is negative. Since m/M effect on the GGE is negative, we thus expect a positive correlation between M and GGE_C and GGE_N respectively. In the case of GGE_P, given the additional direct negative effect of M (Eq. 21), it is difficult to conclude on the overall effect of M without a precise estimate of the correlation between m/M and M on one hand, and of the parameters involved in Eq. 21 on the other hand. It is however safe to conclude that if the correlation is positive, it would be with a shallower slope than for GGE_C and GGE_N.

Figure 1. ln–ln relationship between the ratio of measured-to-asymptotic body weight (m/M) and asymptotic body weight (M), a) for all organisms; b) for unicellular organisms only; c) for invertebrates only; d) for vertebrates only; and e) for multicellular organisms (invertebrates + vertebrates). The lines are the linear regressions. The slope is significantly different from 0 for invertebrates (p=1.04 10^-5) and multicellulars (p<2 10^-16).