Appendix 1

Invasion analysis of the single phenotype competition (SPC) model

For an herbivore species \( j \) to coexist with the other herbivore species in the presence of induction, its per capita growth rate

\[
\frac{1}{H_j} \frac{dH_j}{dt},
\]

must be positive when it is at low densities, \( H_j \approx 0 \). We define \( \hat{I} \) as the equilibrium induction level in the absence of \( H_j \). Thus, for Eq. 1,

\[
\frac{1}{H_j} \frac{dH_j}{dt} = r_j \frac{K_j}{K_j - f_j(\hat{I})}, \quad (A.1)
\]

and

\[
\frac{1}{H_j} \frac{dH_j}{dt} > 0 \quad \text{when} \quad \frac{K_j}{f_j(\hat{I})} < 1. \quad (A.2)
\]

Following Eq. A.2, there are three outcomes possible for Eq. 1,

1) Both herbivores coexist when \( \frac{1}{f_j(\hat{I})} < \frac{K_1}{f_1(\hat{I})} \) and \( \frac{1}{f_j(\hat{I})} < \frac{K_2}{f_2(\hat{I})} \);

2) \( H_j \) excludes \( H_2 \) when \( \frac{1}{f_j(\hat{I})} < \frac{K_1}{f_1(\hat{I})} \) and \( \frac{1}{f_j(\hat{I})} > \frac{K_2}{f_2(\hat{I})} \);

3) \( H_2 \) excludes \( H_j \) when \( \frac{1}{f_j(\hat{I})} > \frac{K_1}{f_1(\hat{I})} \) and \( \frac{1}{f_j(\hat{I})} < \frac{K_2}{f_2(\hat{I})} \).

Note that the fourth combination

\[
\frac{1}{f_j(\hat{I})} > \frac{K_1}{f_1(\hat{I})}, \quad \frac{1}{f_j(\hat{I})} > \frac{K_2}{f_2(\hat{I})},
\]

is not possible as \( \hat{I} \to 0 \) when \( H_j, H_i \to 0 \). Substituting functional forms defined in Eq. 4 into Eq. A.3 above yields, after routine algebra, the criteria defined in Eq. 7.
Appendix 2

Comparison of competition outcomes in the single phenotype competition (SPC) model using different forms of elicitation of inducible changes in plant quality

Below we consider a form of Eq. 1 having differences in both the strength of effect of inducible changes in plant quality on herbivores and in herbivore strengths of elicitation,

\[ f(I) = \gamma I \quad \text{and} \quad \rho_j(H_j, I) = (\alpha_j - \beta_j I)H_j. \quad (B.1) \]

Given similar values for other parameters, the region of parameter space that leads to either coexistence or competitive exclusion differs when herbivores elicit changes in plant quality at different rates \( \alpha_j \), or when that elicitation is inhibited by different self-limitation strengths \( \beta_j \) (Fig. B1). Differences will be larger when elicitation rates, and differences in those rates between herbivores, are large. For both models, the coexistence region shrinks when plant quality self-inhibition shrinks, eventually disappearing as \( \beta_{1,2} \to 0 \). Similar results are obtained as herbivore self-regulation becomes very weak, emphasizing the importance of density-dependent regulation by external factors in allowing coexistence in the SPC model.

![Elicitation same vs. Elicitation different](image)

Figure B1. Examples of equilibrium outcomes of the SPC model as functions of \( \gamma_1 \) and \( \gamma_2 \). Plots in the left column are generated assuming that both herbivore species elicit inducible changes in plant quality at the same per capita rate defined by Eq. 4, while those in the right column are generated assuming that each herbivore species elicits inducible changes in plant quality with a different per capita rate as defined by Eq. B.1. (a) Inhibition of plant quality changes is high, \( \alpha = 50 \) and \( \beta = 5 \) when elicitation is the same for both herbivore species and \( \alpha_1 = 15, \alpha_2 = 100 \) and \( \beta_1 = \beta_2 = 5 \) when elicitation is different. (b) Induction inhibition is low, \( \alpha = 50 \) and \( \beta = 1 \) when elicitation is the same and \( \alpha_1 = 15, \alpha_2 = 100, \beta_1 = 0.5 \) and \( \beta_2 = 2 \) when elicitation is different. Other parameters are the same for all graphs, \( \delta = 0.75, r_1 = 1, r_2 = 1, K_1 = 4 \) and \( K_2 = 2 \).